# Real Polynomials - Practice Test 2

Name: _____ Date: _____

10 questions | 45 marks | 65 minutes

Use identities, remainders and repeated roots to reason about polynomial structure.

A scientific calculator may be used.

**1.** Classify and simplify carefully. [4 marks]

**(a)** Which are polynomials in $x$: $x^3 + \sqrt{2}$, $x^{-1} + 4$, $\frac{x^2-1}{x-1}$ as originally written? [3 marks]

**(b)** Simplify $(2x^2 + 3x - 1) + (x^3 - 5x + 4)$. [1 mark]

**2.** A cubic has zeros $-1, 2, 3$ and $P(0) = 12$. [4 marks]

**(a)** Find its factorised form. [3 marks]

**(b)** Find its leading coefficient and $y$-intercept. [1 mark]

**3.** $P(x) = ax^2 + bx + c$ satisfies $P(0) = 2$, $P(1) = 6$, $P(-1) = 4$. [5 marks]

**(a)** Find $a, b, c$. [3 marks]

**(b)** Explain why these conditions determine a unique polynomial of degree at most 2. [2 marks]

**4.** A division gives $D(x) = x^2 + 1$, quotient $Q(x) = 2x - 3$ and remainder $R(x) = x + 4$. [3 marks]

**(a)** Reconstruct the dividend. [2 marks]

**(b)** Explain why the stated remainder has an allowable degree. [1 mark]

**5.** $P(2)=7$ and $P(-1)=1$. [5 marks]

**(a)** Find the remainder when $P$ is divided by $(x-2)(x+1)$. [3 marks]

**(b)** Can these data determine the whole polynomial? Explain. [2 marks]

**6.** Let $P(x)=x^3-3x^2+4$. [5 marks]

**(a)** Show that 2 is a repeated root. [3 marks]

**(b)** State the sign of $P$ immediately on either side of 2. [2 marks]

**7.** Factor $x^3 + x^2 + x + 1$. [4 marks]

**(a)** Give a complete real factorisation. [2 marks]

**(b)** State all real roots and explain why the quadratic gives none. [2 marks]

**8.** For $f(x) = x^3 - 3x$, use calculus to refine the sketch. [6 marks]

**(a)** Find the stationary points. [3 marks]

**(b)** Classify them and state the real zeros. [3 marks]

**9.** Solve $x^4 + x^2 - 6 = 0$ over the reals. [4 marks]

**(a)** Use a substitution to factor. [2 marks]

**(b)** Explain why there are only two real solutions. [2 marks]

**10.** Consider $P_c(x) = (x-1)^2 + c$. [5 marks]

**(a)** Find the real roots in terms of $c$ when they exist. [3 marks]

**(b)** State the number of distinct real roots for $c < 0$, $c = 0$ and $c > 0$. [2 marks]