Real Polynomials - Practice Test 1

10 questions | 43 marks | 65 minutes

Build from polynomial operations to factor theorems, graphs and a volume application.

Equivalent correct methods earn the corresponding marks.

1. Consider $P(x) = 3x^4 - 2x^3 + 7x - 5$. [3 marks]

(a) State its degree and leading coefficient. [1 mark]

The largest non-zero exponent is 4.

Answer: 4 and 3.

(b) Expand $(x-2)(x^2+x+3)$. [2 marks]

$x^3 + x^2 + 3x - 2x^2 - 2x - 6 = x^3 - x^2 + x - 6$.

Answer: $x^3 - x^2 + x - 6$

2. Let $Q(x) = (x+3)^2(2x-1)$. [4 marks]

(a) State its real roots and their multiplicities. [2 marks]

Set the two factors to zero and read their powers.

Answer: -3 double; $\frac{1}{2}$ simple.

(b) State whether the graph crosses or touches at each root. [2 marks]

Even multiplicity keeps the sign; odd multiplicity changes it.

Answer: Touches at -3; crosses at $\frac{1}{2}$.

3. The identity $(x+1)(ax^2+bx+c) \equiv 2x^3+5x^2+4x+1$ holds. [4 marks]

(a) Find a, b, c. [3 marks]

Compare coefficients: $a=2$, $a+b=5$, $c=1$; check $b+c=4$.

Answer: $a=2, b=3, c=1$.

(b) Explain why substituting one value would not be enough to prove the identity. [1 mark]

Different polynomials can agree at one or several inputs without agreeing everywhere.

Answer: One input checks only one equality.

4. Divide x^3-2x^2-5x+6 by $x-1$. [4 marks]

(a) Find the quotient and remainder. [3 marks]

Synthetic division at 1 gives coefficients $1, -1, -6, 0$.

Answer: $Q=x^2-x-6$, $R=0$.

(b) Hence factor the cubic completely. [1 mark]

$x^2-x-6=(x-3)(x+2)$.

Answer: $(x-1)(x-3)(x+2)$

5. Let $P(x)=2x^3-3x^2+1$. [4 marks]

(a) Find the remainder on division by $x+1$. [2 marks]

$P(-1)=-2-3+1=-4$.

Answer: -4

(b) Find the remainder on division by $2x-1$. [2 marks]

The divisor zero is $\frac{1}{2}$; $P(\frac{1}{2})=\frac{1}{4}-\frac{3}{4}+1=\frac{1}{2}$.

Answer: $\frac{1}{2}$

6. $P(x) = x^3 + kx^2 - 4x - 4$ has factor $x + 1$. [5 marks]

(a) Find k. [2 marks]

$P(-1) = -1 + k + 4 - 4 = k - 1 = 0.$

Answer: $k = 1$

(b) Find all real roots. [3 marks]

$P = x^3 + x^2 - 4x - 4 = (x + 1)(x - 2)(x + 2).$

Answer: $-2, -1, 2$

7. Consider $R(x) = x^4 - 5x^2 + 4$. [4 marks]

(a) Factor R using a substitution. [2 marks]

With $u = x^2$, $u^2 - 5u + 4 = (u - 1)(u - 4)$.

Answer: $(x^2 - 1)(x^2 - 4)$

(b) Solve $R(x) = 0$ over the reals. [2 marks]

Solve the two equations $x^2 = 1$ and $x^2 = 4$.

Answer: $x = -2, -1, 1, 2.$

8. Sketch $y = (x + 1)^2(x - 2)$ without a graphics calculator. [5 marks]

(a) State all intercepts and root multiplicities. [3 marks]

Set factors to zero and calculate $P(0) = -2$.

Answer: X-intercepts -1 double and 2 simple; y-intercept -2.

(b) Describe the two ends and the behaviour at each intercept. [2 marks]

The leading term is positive cubic; the factor powers determine crossing.

Answer: Left down/right up; touches at -1, crosses at 2.

9. A student cancels x from $x^3 = 4x$ and reports two roots. [4 marks]

(a) Explain which solution is lost. [2 marks]

Division by x assumes $x \neq 0$.

Answer: $x = 0$.

(b) Solve correctly. [2 marks]

$x^3 - 4x = x(x-2)(x+2) = 0$.

Answer: $x = -2, 0, 2$.

10. Squares of side x cm are cut from the corners of a 10 cm by 8 cm sheet and the sides are folded into an open box. [6 marks]

(a) Write a volume model and its physical domain. [3 marks]

Height is x and base dimensions are $10 - 2x$, $8 - 2x$, all positive.

Answer: $V = x(10-2x)(8-2x)$; $0 < x < 4$.

(b) For $V = 48$ cm^3, use $x^3 - 9x^2 + 20x - 12 = (x-1)(x-2)(x-6)$ to find the possible cuts. [3 marks]

The volume equation reduces to the supplied cubic. Its roots are $1, 2, 6$; only the first two satisfy $0 < x < 4$.

Answer: $x = 1$ cm or $x = 2$ cm.