Vector calculus - Test 4

Name: _____ Date: _____

10 questions | 64 marks | 90 minutes

Spatial motion, circular motion and the geometry of acceleration. Scientific calculator permitted; no graphing, CAS or numerical-integration software is required. Show working and give exact answers unless a decimal approximation is requested. Angles are in radians. Vector coordinates use the same Cartesian axes throughout each question.

A scientific calculator may be used.

1. A survey drone moves in a straight line at constant velocity from $A = (1, -2, 3)$ m to $B = (7, 2, 3)$ m in 2 seconds, starting at $t = 0$. [5 marks]

(a) Give its position function for the journey. [3 marks]

(b) Find the speed and explain what the zero third component of velocity means. [2 marks]

2. A particle has velocity $\mathbf{v}(t) = (2t, 2, -1)$ m/s and initial position $(0, 1, 4)$ m. [7 marks]

(a) Find its position and acceleration functions. [4 marks]

(b) At $t = 1$, find its speed and a unit vector in its direction of motion. [3 marks]

3. A point on a rotating mechanism has position $\mathbf{r}(t) = (2\cos t, \sin t, 0)$ metres, where t is time in seconds. [6 marks]

(a) Describe its locus and find its velocities at $t = 0$ and $t = \frac{\pi}{2}$. [4 marks]

(b) Explain why a constant rate of change of the parameter does not give constant speed here. [2 marks]

4. A spatial curve is $\mathbf{r}(t) = (t^2, t, t^3)$. [5 marks]

(a) Find all parameter values at which the tangent direction is parallel to the plane $z = 0$. [3 marks]

(b) Write the tangent line there. [2 marks]

5. A particle moves uniformly on a circle according to $\mathbf{r}(t) = \mathbf{c} + (R\cos\omega t, R\sin\omega t)$, where $\mathbf{c}$ is fixed, $R > 0$ and $\omega \neq 0$ are constants. [7 marks]

(a) Prove that $\mathbf{a} = -\omega^2(\mathbf{r} - \mathbf{c})$ and interpret the direction of acceleration. [4 marks]

(b) Express its acceleration magnitude in terms of its speed s and radius R. [3 marks]

6. A helical path has position $\mathbf{r}(t) = (2\cos 3t, 2\sin 3t, t)$ metres, $t \geq 0$ seconds. [6 marks]

(a) Find the time for one turn, the rise per turn and the speed. [4 marks]

(b) Does the complete position function have a positive period? Explain. [2 marks]

7. A particle has position $\mathbf{r}(t) = (t^2, t^2 - 2t, 2t)$ metres for $0 \leq t \leq 2$ seconds. [6 marks]

(a) Find its speed squared as a completed square. [3 marks]

(b) Find the minimum speed and the acceleration when it occurs. Is minimum speed the same as zero acceleration? [3 marks]

8. A curve has position $\mathbf{r}(t) = (\sin t \cos t, \sin^2 t, \cos t)$. [6 marks]

(a) Prove that the curve lies on the unit sphere centred at the origin. [3 marks]

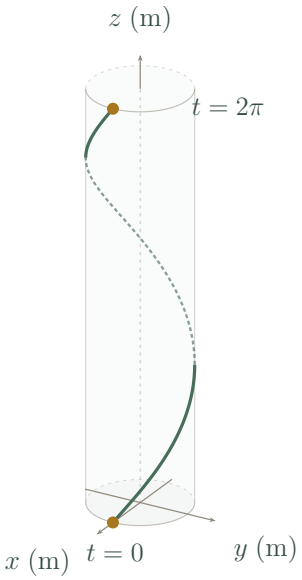
(b) At any point where the derivative is nonzero, prove that the tangent direction is perpendicular to the position vector. [3 marks]

9. A warehouse robot starts at $(0, 0, 0)$ m. For $0 \le t \le 2$ seconds its velocity is $(3, 0, 0)$ m/s. For $2 < t \le 5$ seconds its velocity is $(0, 0, 4)$ m/s. Treat the instantaneous change at $t = 2$ as an idealisation. [6 marks]

(a) Find its final position and total distance travelled. [3 marks]

(b) Compare the magnitude of average velocity with average speed over the full 5 seconds. [3 marks]

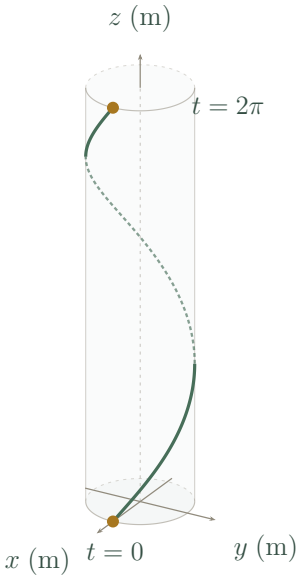
10. A drone follows the helix $\mathbf{r}(t) = (3\cos t, 3\sin t, 4t)$ metres, $0 \leq t \leq 2\pi$, where t is in seconds.
[10 marks]



(a) Find its velocity and speed, and hence the distance travelled during the ascent.
[4 marks]

Question 10 (continued)

A drone follows the helix $\mathbf{r}(t) = (3\cos t, 3\sin t, 4t)$ metres, $0 \le t \le 2\pi$, where t is in seconds.



(b) Find the endpoint separation, then the angle between the velocity and the horizontal plane. Give the angle to the nearest degree. [4 marks]

(c) Explain why the drone has nonzero acceleration even though its speed is constant. [2 marks]