Vector calculus - Test 1

Name: _____ Date: _____

10 questions | 58 marks | 90 minutes

Foundations, interpretation and increasingly connected motion problems. Scientific calculator permitted; no graphing, CAS or numerical-integration software is required. Show working and give exact answers unless a decimal approximation is requested. Angles are in radians. Vector coordinates use the same Cartesian axes throughout each question.

A scientific calculator may be used.

1. A curve is given by $x = t - 1$, $y = 2t^2$, for $0 \leq t \leq 2$. [5 marks]

(a) Eliminate the parameter and state the restriction on the Cartesian equation. [3 marks]

(b) State the starting point, finishing point and direction of tracing. [2 marks]

2. A delivery robot has position $\mathbf{r}(t) = (-2 + 3t, 3 - t)$ metres, where $t \geq 0$ is time in seconds. A painted line has equation $y = 1$. [6 marks]

(a) Find when and where the robot crosses the painted line. [3 marks]

(b) Find its speed and the distance travelled before the crossing. [3 marks]

3. A particle has position $\mathbf{r}(t) = (t^2, 3t - t^2)$ metres for $0 \leq t \leq 3$, with t in seconds. [6 marks]

(a) Find the velocity and acceleration functions. [2 marks]

(b) Find the greatest height above the x-axis and explain why the particle is not at rest there. [4 marks]

4. A particle has velocity $\mathbf{v}(t) = (4t, 3)$ m/s and initial position $\mathbf{r}(0) = (-1, 2)$ m. [6 marks]

(a) Find its position function. [3 marks]

(b) Find the displacement and its magnitude from $t = 0$ to $t = 2$ seconds. [3 marks]

5. A quadratic Bézier curve has control points $P_0 = (0, 0)$, $P_1 = (1, 2)$, $P_2 = (4, 0)$. Its rule is $\mathbf{B}(u) = (1-u)^2 P_0 + 2u(1-u)P_1 + u^2 P_2$ where $0 \le u \le 1$. [6 marks]

(a) Find $\mathbf{B}(\frac{1}{2})$ and determine whether it equals P_1. [3 marks]

(b) Find the derivative vectors at both endpoints. [3 marks]

6. A curve has coordinates $x = t^2 + 1$, $y = t^3 - t$. [5 marks]

(a) Find the point and tangent gradient at $t = 1$. [3 marks]

(b) Find a Cartesian equation of that tangent. [2 marks]

7. A particle follows $\mathbf{r}(t) = (t^2, 2t)$ metres for $t \geq 0$ seconds. [5 marks]

(a) Find its speed function. [2 marks]

(b) Find the rate of change of speed at $t = 1$. State whether it is speeding up then. [3 marks]

8. A particle has position $\mathbf{r}(t) = (1 + 3\cos(2t),\ -2 + 3\sin(2t))$ metres, with t in seconds. [6 marks]

(a) Describe the circular path, its direction of motion and the time for one revolution. [4 marks]

(b) Find its speed. [2 marks]

9. A curve is parameterised by $\mathbf{r}(t) = (t^2, \frac{2}{3}t^3)$, $0 \le t \le 1$. [6 marks]

(a) Show that its arc-length integrand simplifies to a function without an absolute-value sign on this interval. [3 marks]

(b) Find the exact length of the curve. [3 marks]

10. A camera moves above a horizontal stage with position $\mathbf{r}(t) = (2t, 6t - t^2)$ metres, $0 \leq t \leq 6$, where t is measured in seconds. [7 marks]

(a) Find when the camera reaches its greatest height, and find its position then. [3 marks]

(b) Prove that the camera has its minimum speed at that same time, and find this speed. [4 marks]