Arc length in two and three dimensions

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find length of $(5t, 12t)$ on $0 \leq t \leq 1$. [2 marks]

Speed is $\sqrt{25+144}$.

Integrate 13 over an interval of length 1.

Answer: 13

2. Find length of $(3\cos t, 3\sin t)$ on $0 \leq t \leq \frac{\pi}{2}$. [2 marks]

The speed is 3.

Multiply by the parameter interval length.

Answer: $\frac{3\pi}{2}$

3. Find length of $(\cos t, \sin t, 2t)$ on $0 \leq t \leq 2\pi$. [2 marks]

Speed squared is 5.

Integrate constant speed $\sqrt{5}$.

Answer: $2\pi\sqrt{5}$

4. Set up length for (t, t^2) on $0 \le t \le 1$ without evaluating. [2 marks]

The derivative is $(1, 2t)$.

Take its magnitude before integration.

Answer: $\int_0^1 \sqrt{1+4t^2}\, dt$

5. Find length of $(t^2, \frac{2}{3}t^3)$ on $0 \le t \le 1$. [2 marks]

Speed is $2t\sqrt{1+t^2}$ for nonnegative t.

Use $u = 1 + t^2$, so the integral becomes $\int_1^2 \sqrt{u}\, du$.

Answer: $\frac{2}{3}(2\sqrt{2} - 1)$

6. For $(\cos t, \sin t)$ on $0 \le t \le 4\pi$, find distance and displacement. [2 marks]

Speed 1 is integrated over interval length 4π.

The path makes two full turns back to its start.

Answer: Distance 4π; displacement $(0, 0)$.