Sine/cosine parameterisations and circular motion

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find the Cartesian locus of $(3\cos t, 2\sin t)$. [2 marks]

Divide by the amplitudes, square and add.

Use the Pythagorean trig identity.

Answer: $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

2. Find speed and acceleration magnitude for $(5\cos 2t, 5\sin 2t)$. [2 marks]

Use radius times angular speed for speed.

Use radius times angular speed squared for acceleration.

Answer: 10 and 20.

3. At 0, describe direction for $(2\cos t, -2\sin t)$. [2 marks]

The derivative at zero is $(0, -2)$.

That is downward at the rightmost point.

Answer: Down from $(2, 0)$, clockwise.

4. Find centre and radius of $(1+4\cos t,\ -2+4\sin t)$. [2 marks]

The constants shift the centre.

The equal amplitudes give the radius.

Answer: Centre $(1,\ -2)$, radius 4.

5. Find speed for $(2\cos t, 2\sin t, t)$. [2 marks]

Velocity is $(-2\sin t, 2\cos t, 1)$.

Squared speed is $4+1$.

Answer: $\sqrt{5}$

6. Compare speeds at 0 and $\frac{\pi}{2}$ for $(4\cos t, 2\sin t)$. [2 marks]

Velocity is $(-4\sin t, 2\cos t)$.

The specified times give $(0, 2)$ and $(-4, 0)$.

Answer: 2 and 4.

