Parametric gradients and tangent directions

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\frac{dy}{dx}$ for $x = t^2 + 1, y = 3t$, t nonzero. [2 marks]

Derivatives are $2t$ and 3.

Divide y rate by x rate.

Answer: $\frac{3}{2t}$

2. Find the tangent at 2 for $x = t, y = t^3$. [2 marks]

The point is $(2, 8)$.

Gradient is $3t^2 = 12$.

Answer: $y - 8 = 12(x - 2)$.

3. Find the tangent at 0 for $(\cos t, \sin t)$. [2 marks]

The point is $(1, 0)$.

The derivative is $(0, 1)$, giving a vertical line.

Answer: $x = 1$.

4. Use a limit to find the tangent at 0 for $x = t^3, y = t^2$. [2 marks]

For nonzero t, $\frac{dx}{dy} = \frac{3t}{2}$.

This tends to 0, giving a vertical tangent.

Answer: $x = 0$.

5. Give a vector tangent equation at 1 for (t, t^2, t^3). [2 marks]

The point is $(1, 1, 1)$.

The derivative there is $(1, 2, 3)$.

Answer: $\mathbf{x} = (1, 1, 1) + s(1, 2, 3)$.

6. Find the horizontal tangent of $x = 2t, y = t^2 - 4t$. [2 marks]

$\frac{dy}{dt} = 2t - 4 = 0$ gives 2; $\frac{dx}{dt} = 2$.

The corresponding point is $(4, -4)$.

Answer: $y = -4$.