Differentiating and integrating components

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Differentiate $(t^4, \ln t)$ for $t > 0$. [2 marks]

Use the power rule.

The logarithmic coordinate requires a positive domain.

Answer: $(4t^3, \frac{1}{t})$

2. Find the second derivative of $(t^3, 2t^2, \cos t)$. [2 marks]

The first derivative is $(3t^2, 4t, -\sin t)$.

Differentiate again.

Answer: $(6t, 4, -\cos t)$

3. Integrate $(3t^2, 2t, 1)$ with initial position $(0, 1, -1)$ at zero. [2 marks]

Integrate each coordinate.

Use one constant for each initial coordinate.

Answer: $(t^3, t^2 + 1, t - 1)$

4. Evaluate $\int_0^2 (t, 2t, 3)\,dt$. [2 marks]

Antiderivative $(\frac{t^2}{2}, t^2, 3t)$.

Evaluate upper minus lower.

Answer: $(2, 4, 6)$

5. Given $\mathbf{v} = (e^t, \cos t)$ and $\mathbf{r}(0) = (2, 0)$, find position. [2 marks]

Integrate to $(e^t + C_1, \sin t + C_2)$.

Initial values give $C_1 = 1, C_2 = 0$.

Answer: $(e^t + 1, \sin t)$

6. Evaluate $\int_0^{\frac{\pi}{2}} (\cos t, \sin t)\,dt$. [2 marks]

Use antiderivative $(\sin t, -\cos t)$.

Subtract values at both endpoints.

Answer: $(1, 1)$