Rates of change and differential equations - Test 6

Name: _____ Date: _____

10 questions | 63 marks | 75 minutes

Combine methods and critique common modelling errors.

A scientific calculator may be used.

1. The curve $x^2y + xy^2 = 6$ passes through $(1, 2)$. [6 marks]

(a) Find $\frac{dy}{dx}$. [3 marks]

(b) Find the tangent at the stated point. [3 marks]

2. A rectangle has side x and perimeter 30 cm. Its x side increases at 1 cm/s. At $x = 4$ cm: [6 marks]

(a) Write a relation suitable for differentiation. [3 marks]

(b) Find the area rate in square centimetres per second. [3 marks]

3. Two proposed solutions are $f(x) = 2e^x$ and $g(x) = e^{2x}$. The initial-value problem is $y' = 2y$, $y(0) = 2$. [6 marks]

(a) Explain which requirement each proposed function fails. [3 marks]

(b) Find the correct solution and verify both requirements. [3 marks]

4. The equation is $y' = 1 - y^2$. A numerical sketch begins at $y(0) = 0$. [6 marks]

(a) Identify and classify the constant equilibria using the slope signs. [3 marks]

(b) A sketch crosses $y = 1$ with positive slope. Explain the error, and verify the proposed exact solution $y = \frac{e^{2x} - 1}{e^{2x} + 1}$. [3 marks]

5. A particle has acceleration $a(t) = 6t - 6$ m/s^2, $v(0) = 0$ and $x(0) = 5$ m for $0 \le t \le 3$ s. [6 marks]

(a) Find its velocity and position. [3 marks]

(b) Find its displacement and total distance during the three seconds. [3 marks]

6. Consider $y' = \frac{y}{x+1}$, $y(0) = -2$, $x > -1$. [6 marks]

(a) Find the solution and its interval containing the initial point. [3 marks]

(b) Identify any constant solution lost during separation, or explain why there is none. [3 marks]

7. A reservoir contains V litres and satisfies $V' = 4 - 0.1V$, $V(0) = 10$, with t in minutes. The inflow is 4 L/min and the outflow rate is $0.1V$ L/min. [6 marks]

(a) Find the amount model. [3 marks]

(b) Find the first time the volume reaches 30 L. If the reservoir capacity is 35 L, explain whether this unchanged model can remain valid for all nonnegative time. [3 marks]

8. A population satisfies $P^{'} = 0.2P(1 - \frac{P}{900})$ with $P(0) = 600$. Here t is in days, P is in individuals, and t is at least zero. [9 marks]

(a) Find $P(t)$ and the carrying capacity. [3 marks]

(b) Find the global largest possible positive growth rate and when this trajectory reaches its associated amount. [3 marks]

(c) For this initial population, find the greatest growth rate for t at least zero. [3 marks]

9. A displacement model is $x = 2\cos 3t - 2\sin 3t$. Here t is measured in seconds and displacement x in metres. [6 marks]

(a) Differentiate the given displacement to find velocity and acceleration, and verify that $a = -9x$. [3 marks]

(b) Find the amplitude and maximum speed. [3 marks]

10. For $y' = x - y$, a student calls $y = x$ an equilibrium because its field slopes vanish. [6 marks]

(a) Identify and correct the error. [3 marks]

(b) Prove that no constant equilibrium exists. [3 marks]