Rates of change and differential equations - Test 5

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Apply calculus while checking signs, distances and assumptions.

A scientific calculator may be used.

1. The curve $x^3 + y^3 = 9$ passes through $(1, 2)$. [6 marks]

(a) Find $\frac{dy}{dx}$. [3 marks]

(b) Find the tangent at the stated point. [3 marks]

2. Water leaves a cylinder of fixed radius 3 m at 6 m^3/min. Let h be its depth. [6 marks]

(a) Write a relation suitable for differentiation. [3 marks]

(b) Find the depth rate in metres per minute. [3 marks]

3. A student claims the general solution of $y' = 3y$ is $y = e^{3x + C}$ and therefore the initial condition $y(0) = -2$ is impossible. [6 marks]

(a) Explain the missing step and give the full solution family. [3 marks]

(b) Find and verify the solution with $y(0) = -2$. [3 marks]

4. Consider the slope field for $y' = -y^3$. [6 marks]

(a) Find slopes at $(0, -1)$ and $(2, 2)$. [3 marks]

(b) Find and classify constant equilibria, or explain why none exist. [3 marks]

5. Solve the initial-value problem $y' = \frac{2x}{1+x^2}$, $y(0) = 3$. [6 marks]

(a) Find $y(x)$. [3 marks]

(b) Find $y(1)$ and verify your derivative. [3 marks]

6. Consider $y' = 2x(y-1)$ with $y(0) = -1$. [6 marks]

(a) Solve the initial-value problem and state its domain. [3 marks]

(b) Find the equilibrium lost in separation. Explain whether this initial-value solution ever reaches it. [3 marks]

7. A medicine concentration follows exponential decay. It starts at 12 mg/L and is 3 mg/L after 8 h. [6 marks]

(a) Find the concentration model. [3 marks]

(b) Find the half-life. [3 marks]

8. A population follows $P' = 0.4P(1 - \frac{P}{1000})$, $P(0) = 1200$, with t in years and $t \geq 0$. [6 marks]

(a) Find the model, initial rate and long-term limit. [3 marks]

(b) A student says the population must first rise to 1500 before settling at 1000. Explain why the differential equation rules this out. [3 marks]

9. A particle moves according to $x'' = -4x$, with $x(0) = 3$ m and $v(0) = 8$ m/s. [6 marks]

(a) Find $x(t)$ and the amplitude. [3 marks]

(b) Find the maximum speed and the first positive time at which the particle reaches its positive extreme position. [3 marks]

10. A particle has velocity $v = t - 1$ m/s for $0 \le t \le 3$ s. A student calls $\int_0^3 v\,dt$ the total distance. [6 marks]

(a) Identify and correct the error. [3 marks]

(b) Find both displacement and total distance on the stated interval. [3 marks]