Rates of change and differential equations - Test 4

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Reason from functions and slope information.

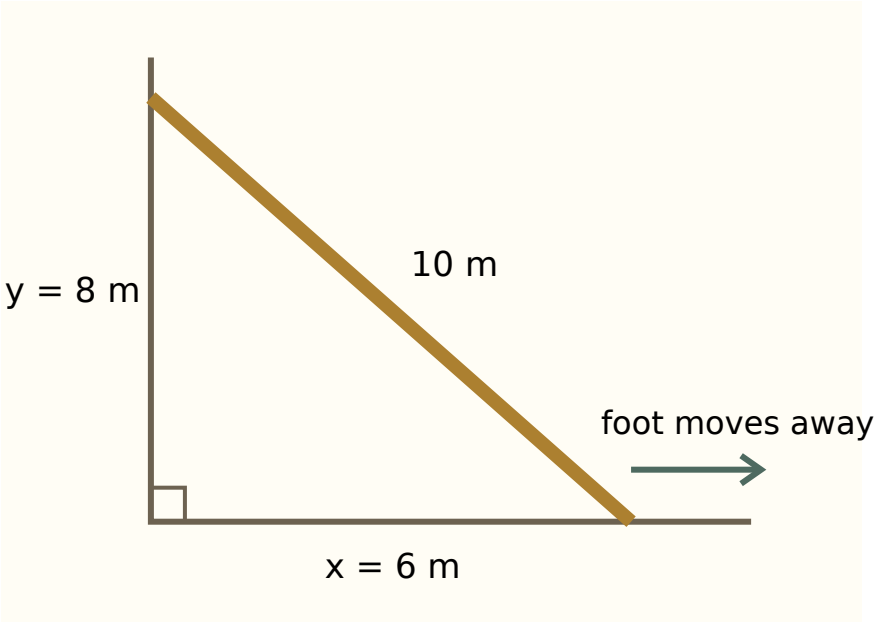
A scientific calculator may be used.

1. The relation $\sin y = x$ passes through $(0, 0)$. [6 marks]

(a) Find $\frac{dy}{dx}$. [3 marks]

(b) Find the tangent at the stated point. [3 marks]

2. A 10 m ladder has its foot 6 m from a wall and moving away at 0.4 m/s. Let y be top height. [6 marks]



(a) Write a relation suitable for differentiation. [3 marks]

(b) Find the ladder-top height rate in metres per second. [3 marks]

3. A proposed initial-value solution is $y = \frac{2}{1-2x}$, $x < \frac{1}{2}$. [6 marks]

(a) Verify $y' = y^2$. [3 marks]

(b) Verify $y(0) = 2$. Explain why both checks are needed. [3 marks]

4. An autonomous differential equation has the form $y' = a - by$. Its slope is 6 at $y = 0$ and 2 at $y = 2$. [6 marks]

(a) Determine a and b, and find the constant equilibrium. [3 marks]

(b) Classify the equilibrium and solve the equation with $y(0) = 1$. [3 marks]

5. A tank starts with 20 L. Its net inflow rate is $q(t) = 4 - t$ L/min for $0 \le t \le 4$ and $q(t) = -2$ L/min for $4 < t \le 7$. [6 marks]

(a) Find the amount at 4 minutes and at 7 minutes. [3 marks]

(b) Find the maximum amount during the stated interval and explain why it occurs there. [3 marks]

6. Consider $y' = (1 + y^2)$, $y(0) = 0$. [6 marks]

(a) Find the solution and its interval containing the initial point. [3 marks]

(b) Identify any constant solution lost during separation, or explain why there is none. [3 marks]

7. A laboratory compares two amount models for $t \geq 0$ days: $A(t) = 100e^{0.1t}$ and $B(t) = 100 + 10t$. [6 marks]

(a) Show that they have the same initial amount and the same initial rate. Find each amount at 10 days. [3 marks]

(b) Explain why equal initial values and rates do not make these models identical. [3 marks]

8. A logistic population satisfies $P' = rP(1 - \frac{P}{1000})$. When $P = 200$, its rate is 32 individuals/day. Initially $P(0) = 100$. [6 marks]

(a) Find r and the population model. [3 marks]

(b) Find the largest growth rate and the time it occurs. [3 marks]

9. A displacement model is $x'' = -16(x-2)$, $x(0)=3$, $v(0)=0$. Here t is measured in seconds and displacement x in metres. [6 marks]

(a) Find or verify the displacement and its derivative relation. [3 marks]

(b) Find the equilibrium position and period. [3 marks]

10. A 10 m ladder slides with its foot at distance x from a wall and its top at height y. A student substitutes $x=6$, $y=8$ into $x^2+y^2=100$ before differentiating. [6 marks]

(a) Identify and correct the error. [3 marks]

(b) At that instant, find the top rate if the foot moves away at 0.2 m/s. [3 marks]