Rates of change and differential equations - Test 3

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Connect geometry, separation and rate models.

A scientific calculator may be used.

1. The relation $xy + y^2 = 6$ passes through $(1, 2)$. [6 marks]

(a) Find $\frac{dy}{dx}$. [3 marks]

(b) Find the tangent at the stated point. [3 marks]

2. A conical tank has its vertex at the bottom and radius-to-height ratio $r = \frac{h}{2}$. Water enters at 3 m^3/min. At depth $h = 2$ m: [6 marks]

(a) Write a relation suitable for differentiation. [3 marks]

(b) Find the water-depth rate in metres per minute. [3 marks]

3. The table shows values of a differentiable function satisfying $y' = 2x$. [6 marks]

Observed values

x	y
0	3
1	4
2	7
3	12

(a) Use the initial value in the table to find the function. [3 marks]

(b) Check whether all the table entries are consistent with this model. Does agreement with a finite table alone prove a differential equation for all real x? [3 marks]

4. Consider the slope field for $y' = y(y-2)$. [6 marks]

(a) Find slopes at $(0, 1)$ and $(1, 3)$. [3 marks]

(b) Find and classify constant equilibria, or explain why none exist. [3 marks]

5. Solve the initial-value problem $y' = \frac{3}{x+1}$, $y(0) = 2$, $x > -1$. [6 marks]

(a) Find $y(x)$. [3 marks]

(b) Find $y(e-1)$ and verify your derivative. [3 marks]

6. Consider $y' = \frac{x}{y}$, $y(0) = 2$. [6 marks]

(a) Find the solution and its interval containing the initial point. [3 marks]

(b) Identify any constant solution lost during separation, or explain why there is none. [3 marks]

7. An object cools according to $T' = -0.2(T - 20)$, with $T(0) = 80$. Temperature is in degrees Celsius and time in minutes. [6 marks]

(a) Find the temperature model. [3 marks]

(b) Find when the temperature first reaches 40 degrees Celsius. [3 marks]

8. A population satisfies $P^{'} = 0.3P(1 - \frac{P}{800})$ with $P(0) = 200$. Here t is in days, P is in individuals, and t is at least zero. [6 marks]

(a) Find $P(t)$ and the carrying capacity. [3 marks]

(b) Find the global largest possible positive growth rate and when this trajectory reaches its associated amount. [3 marks]

9. A displacement model is $x'' = -x$, $x(0) = 3$, $v(0) = 4$. Here t is measured in seconds and displacement x in metres. [6 marks]

(a) Find or verify the displacement and its derivative relation. [3 marks]

(b) Find the amplitude and period. [3 marks]

10. A logistic rate is $P' = 0.5P(1 - \frac{P}{400})$. A student says growth is greatest at $P = 400$. [6 marks]

(a) Identify and correct the error. [3 marks]

(b) Calculate the greatest possible growth rate. [3 marks]