Rates of change and differential equations - Test 2

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Use different equation forms and interpret their solutions.

A scientific calculator may be used.

1. The ellipse $x^2 + 4y^2 = 20$ passes through $(2, 2)$. [6 marks]

(a) Find $\frac{dy}{dx}$. [3 marks]

(b) Find the tangent at the stated point. [3 marks]

2. A sphere radius r decreases at 0.1 cm/s. At $r = 3$ cm: [6 marks]

(a) Write a relation suitable for differentiation. [3 marks]

(b) Find the volume rate in cubic centimetres per second and interpret its sign. [3 marks]

3. A proposed initial-value solution is $y = x^3 + 4$. [6 marks]

(a) Verify $y' = 3x^2$. [3 marks]

(b) Verify $y(1) = 5$. Explain why both checks are needed. [3 marks]

4. Consider the slope field for $y' = 3 - y$. [6 marks]

(a) Find slopes at $(0, 1)$ and $(2, 4)$. [3 marks]

(b) Find and classify constant equilibria, or explain why none exist. [3 marks]

5. Solve the initial-value problem $y' = 4e^{2x}$, $y(0) = 5$. [6 marks]

(a) Find $y(x)$. [3 marks]

(b) Find $y(\ln 2)$ and verify your derivative. [3 marks]

6. Consider $y' = y^2$, $y(0) = 1$. [6 marks]

(a) Find the solution and its interval containing the initial point. [3 marks]

(b) Identify any constant solution lost during separation, or explain why there is none. [3 marks]

7. A sample has half-life 6 h and initial mass 90 g. [6 marks]

(a) Find the amount model. [3 marks]

(b) Find the mass after 12 hours. [3 marks]

8. A population satisfies $P^{'} = 0.2P(1 - \frac{P}{600})$ with $P(0) = 100$. Here t is in days, P is in individuals, and t is at least zero. [6 marks]

(a) Find $P(t)$ and the carrying capacity. [3 marks]

(b) Find the global largest possible positive growth rate and when this trajectory reaches its associated amount. [3 marks]

9. A displacement model is $x'' = -9x$, $x(0) = 0$, $v(0) = 6$. Here t is measured in seconds and displacement x in metres. [6 marks]

(a) Find or verify the displacement and its derivative relation. [3 marks]

(b) Find the period and maximum speed. [3 marks]

10. A student solves $y' = y^2$, $y(0) = 1$ and reports $y = \frac{1}{1 - x}$ for every real x. [6 marks]

(a) Identify and correct the error. [3 marks]

(b) Describe the behaviour as x approaches 1 from below. [3 marks]