Rates of change and differential equations - Test 1

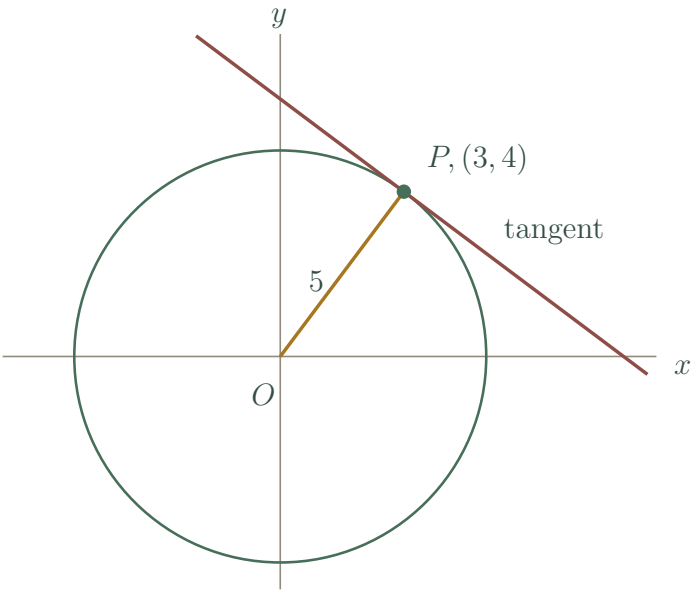
Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Build fluency across implicit rates, initial conditions and models.

A scientific calculator may be used.

1. The circle $x^2 + y^2 = 25$ passes through $(3, 4)$. [6 marks]



(a) Find $\frac{dy}{dx}$. [3 marks]

(b) Find the tangent at the stated point. [3 marks]

2. A square side s increases at 2 cm/s. At $s = 5$ cm: [6 marks]

(a) Write a relation suitable for differentiation. [3 marks]

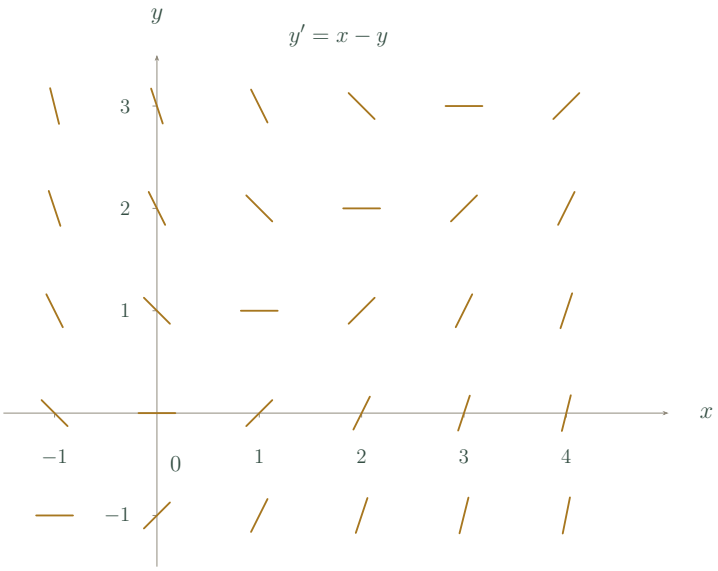
(b) Find the area rate in cm squared per second. [3 marks]

3. A proposed initial-value solution is $y = 3e^{-2x}$. [6 marks]

(a) Verify $y' = -2y$. [3 marks]

(b) Verify $y(0) = 3$. Explain why both checks are needed. [3 marks]

4. Consider the slope field for $y' = x - y$. [6 marks]



(a) Find slopes at $(-1, 1)$ and $(1, 0)$. [3 marks]

(b) Identify the line of zero slopes and explain whether it is a constant equilibrium. [3 marks]

5. Solve the initial-value problem $y' = 6x^2 - 2,\ y(0) = 4$. [6 marks]

(a) Find $y(x)$. [3 marks]

(b) Find $y(2)$ and verify your derivative. [3 marks]

6. Consider $y' = 2xy,\ y(0) = 3$. [6 marks]

(a) Find the solution and its interval containing the initial point. [3 marks]

(b) Identify any constant solution lost during separation, or explain why there is none. [3 marks]

7. A culture grows from 200 to 300 individuals in 4 days under $P' = kP$, with constant k. [6 marks]

(a) Find the amount model. [3 marks]

(b) Find the doubling time. [3 marks]

8. A population satisfies $P' = 0.4P(1 - \frac{P}{1000})$ with $P(0) = 200$. Here t is in days, P is in individuals, and t is at least zero. [6 marks]

(a) Find $P(t)$ and the carrying capacity. [3 marks]

(b) Find the global largest possible positive growth rate and when this trajectory reaches its associated amount. [3 marks]

9. A displacement model is $x'' = -4x$, $x(0) = 3$, $v(0) = 0$. Here t is measured in seconds and displacement x in metres. [6 marks]

(a) Find or verify the displacement and its derivative relation. [3 marks]

(b) Find the period and maximum speed. [3 marks]

10. A student separates $y' = 2y$ and writes $\ln|y| = 2x + C$, then claims that y must be positive. [6 marks]

(a) Identify and correct the error. [3 marks]

(b) Find a solution meeting $y(0) = -3$ and verify it. [3 marks]