# Exponential and logistic models

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

**1.** An amount decays with half-life 5 hours and starts at 80 g. Find the exponential model. [2 marks]

Half-life means $e^{5k} = \frac{1}{2}$.

$k = -\frac{\ln 2}{5}$ per hour.

**Answer:** $M = 80e^{-\frac{(\ln 2)t}{5}}$

**2.** $N = 120e^{0.08t}$, $t$ in days. Find the time to reach 300. [2 marks]

Divide by 120 and take logarithms.

$0.08t = \ln 2.5$.

**Answer:** $t = \frac{\ln(2.5)}{0.08} \approx 11.45$ days.

**3.** For $P' = 0.3P(1 - \frac{P}{800})$ and $P(0) = 100$, find $P(t)$. [2 marks]

$A = \frac{800 - 100}{100} = 7$.

Substitute $r = 0.3$ and $K = 800$.

**Answer:** $P = \frac{800}{1 + 7e^{-0.3t}}$

**4.** For the preceding logistic model, find the maximum growth rate. [2 marks]

The maximum occurs at half the carrying capacity.

$0.3(400)(1-\frac{400}{800})=60.$

**Answer:** $60$ units/time at $P=400.$

**5.** $P=\frac{500}{1+9e^{-0.2t}}$. Find the initial amount and long-term limit. [2 marks]

At zero the denominator is $10.$

As $t$ increases, the exponential term tends to zero.

**Answer:** $P(0)=50$; limit $500.$

**6.** Under $P^{'}=0.4P(1-\frac{P}{1000})$, state the predicted direction of change at $P=1200.$ [2 marks]

$P^{'}=0.4(1200)(1-1.2)=-96.$

The negative sign indicates decrease.

**Answer:** Decreasing, at $96$ units/time.