Separation of variables

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $y' = 4xy$ with $y(0) = 2$. [2 marks]

$\frac{dy}{y} = 4x\,dx$ gives $\ln|y| = 2x^2 + C$.

Initial data gives multiplier 2.

Answer: $y = 2e^{2x^2}$

2. Solve $y' = \frac{x}{y}$ with $y(0) = 3$. [2 marks]

$y\,dy = x\,dx$ gives $y^2 = x^2 + C$.

$C = 9$; the positive branch matches $y(0) = 3$.

Answer: $y = \sqrt{x^2 + 9}$

3. Solve $y' = (1 + y^2)$ with $y(0) = 0$. State the interval around zero. [2 marks]

$\frac{dy}{1+y^2} = dx$ gives $\arctan y = x + C$.

$C = 0$ and tangent is finite only between its adjacent poles.

Answer: $y = \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

4. Find all constant solutions of $y' = x(y-2)(y+1)$. [2 marks]

A constant solution has zero derivative.

Both factors give values making the right side zero for every x.

Answer: $y = 2$ and $y = -1$.

5. Solve $y' = -y^2$ with $y(0) = 1$. [2 marks]

$y^{-2}dy = -dx$ gives $-\frac{1}{y} = -x + C$.

$C = -1$, giving the result and domain.

Answer: $y = \frac{1}{x+1}$ on $x > -1$.

6. Solve $y' = \frac{y}{x}$ with $y(1) = -2$, on $x > 0$. [2 marks]

$\frac{dy}{y} = \frac{dx}{x}$ gives $\ln|y| = \ln x + C$.

$y = Ax$ and $A = -2$; retaining a signed multiplier is essential.

Answer: $y = -2x$