Direct integration

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $y' = 5x^4 - 2$ with $y(1) = 3$. [2 marks]

Integrate to $x^5 - 2x + C$.

$3 = 1 - 2 + C$, so $C = 4$.

Answer: $y = x^5 - 2x + 4$

2. Solve $y' = 6e^{2x}$ with $y(0) = 7$. [2 marks]

The antiderivative is $3e^{2x} + C$.

$7 = 3 + C$ gives $C = 4$.

Answer: $y = 3e^{2x} + 4$

3. Solve $y' = \frac{2}{x}$ with $y(1) = 4$, $x > 0$. [2 marks]

Integrate on the positive domain.

$\ln 1 = 0$, so the constant is 4.

Answer: $y = 2\ln x + 4$

4. A tank initially contains 20 L. The net inflow is $3t + 2$ L/min for $0 \le t \le 4$. Find its amount at 4 min. [2 marks]

The increase is $\int_0^4 (3t + 2)dt = [1.5t^2 + 2t]_0^4 = 32$ L.

Add the initial 20 L.

Answer: 52 L

5. A particle has $a = 4$, $v(0) = -3$, $s(0) = 2$. Find $s(t)$. [2 marks]

Integrate: $v = 4t - 3$.

Integrate again and use $s(0) = 2$.

Answer: $s = 2t^2 - 3t + 2$

6. A particle has $v(t) = t - 2$ m/s on $0 \le t \le 4$. Find displacement and total distance. [2 marks]

$\int_0^4 (t - 2)dt = 0$.

The velocity changes sign at $t = 2$. The two triangular speed areas are each 2 m.

Answer: Displacement 0 m; total distance 4 m.