Implicit differentiation

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\frac{dy}{dx}$ for $x^2 + 3y^2 = 12$. [2 marks]

$2x + 6yy' = 0.$

Divide by $6y$, where $y \neq 0.$

Answer: $-\frac{x}{3y}$

2. Find $\frac{dy}{dx}$ for $x^3 + y^3 = 9$. [2 marks]

$3x^2 + 3y^2y' = 0.$

$y' = -\frac{x^2}{y^2}$, for $y \neq 0.$

Answer: $-\frac{x^2}{y^2}$

3. Find the tangent to $x^2 + y^2 = 10$ at $(1, 3)$. [2 marks]

The point satisfies $1 + 9 = 10.$

$y' = -\frac{x}{y} = -\frac{1}{3}$ at the point.

Answer: $y - 3 = -\frac{1}{3}(x - 1)$

4. Differentiate $x^2 + xy + y^2 = 7$ implicitly. [2 marks]

$2x + y + xy' + 2yy' = 0$.

Collect $y'(x + 2y) = -(2x + y)$.

Answer: $y' = -\frac{2x+y}{x+2y}$

5. For $x^2 + y^2 = 9$, identify points where the tangent is vertical. [2 marks]

$y' = -\frac{x}{y}$ has a zero denominator at $y = 0$.

The circle then gives $x = \pm 3$; these are its left and right endpoints.

Answer: $(3, 0)$ and $(-3, 0)$

6. Find the gradient of $\sin y = x$ at $(0, 0)$. [2 marks]

$\cos y\, y' = 1$.

At $(0, 0)$, $\cos 0 = 1$, so $y' = 1$.

Answer: 1