Integration techniques and applications - practice test 6

Name: _____ Date: _____

10 questions | 64 marks | 75 minutes

Challenging combined-method problems, parameter reasoning, singularities and geometric models.

A scientific calculator may be used.

1. For $x > 0$, consider $\int x^n \ln x \, dx$, where n is real. [7 marks]

(a) For $n \neq -1$, derive a general antiderivative using parts. [4 marks]

(b) Handle $n = -1$ separately and evaluate $\int_1^{e^2} \frac{\ln x}{x} \, dx$. [3 marks]

2. Let $I = \int_0^1 \frac{x^3}{1+x^2}\,dx$. [6 marks]

(a) Use polynomial division to find an antiderivative. [3 marks]

(b) Evaluate I and show how the single substitution $u = 1 + x^2$ gives the same result. [3 marks]

3. Consider $I = \int_0^{\frac{\pi}{2}} \cos^4 x\,dx$. [5 marks]

(a) Derive an identity expressing cosine to the fourth as a sum of cosines. [3 marks]

(b) Evaluate I exactly. [2 marks]

4. Consider $f(x)=\frac{x^2+2}{x^3-x}$. [8 marks]

(a) Find its partial-fraction decomposition. [3 marks]

(b) Evaluate its integral from 2 to 3 exactly. [3 marks]

(c) Explain why the same ordinary endpoint calculation cannot be used from 1 to 3. [2 marks]

5. Seek an antiderivative of $e^{2x}\cos x$ in the form $F(x) = e^{2x}(A\cos x + B\sin x)$. [6 marks]

(a) Differentiate the proposed form and find A and B. [3 marks]

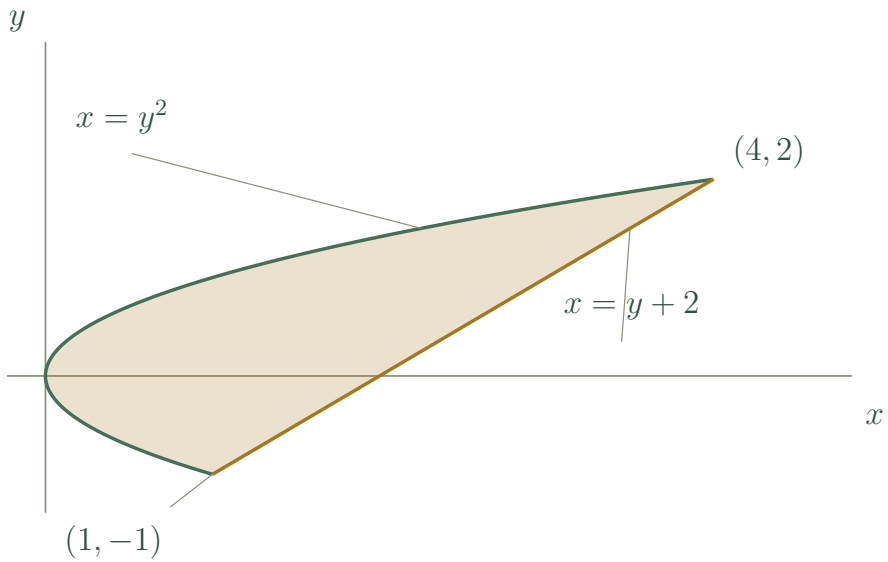
(b) Evaluate $\int_0^{\pi} e^{2x}\cos x\,dx$. [3 marks]

6. Let $I = \int_0^{\ln 2} \frac{e^x}{\sqrt{1+e^x}}\,dx$. [5 marks]

(a) Choose a substitution, change the bounds and evaluate I. [3 marks]

(b) Find the average value of the integrand on the original interval. [2 marks]

7. The region bounded by $x = y^2$ and $x = y + 2$ is rotated about the y axis. [7 marks]



(a) Find the two intersection y values and the plane area. [3 marks]

(b) Find the exact solid volume using washers. [4 marks]

8. The region $0 \leq x \leq \pi$, $0 \leq y \leq \sin x$ is rotated about the x axis. [6 marks]

(a) Find the total volume. [3 marks]

(b) Find the volume from $x = 0$ to a, where $0 \leq a \leq \pi$, and show that the plane $x = \frac{\pi}{2}$ divides the solid into equal volumes. [3 marks]

9. A tank starts with 30 L. During $0 \le t \le 4$ minutes, inflow is $6t(4-t)$ L/min and outflow is a constant 8 L/min. [7 marks]

(a) Find the total inflow, total outflow and final amount. [3 marks]

(b) Find an amount function $V(t)$, including its initial amount. [2 marks]

(c) Explain why the amount initially decreases even though the final amount exceeds 30 L. [2 marks]

10. For $a > 0$, the region under $y = ax(1 - x)$ on $0 \leq x \leq 1$ has area A. Its rotation about the x axis has volume V. [7 marks]

(a) Find A and V in terms of a. [4 marks]

(b) Eliminate a to express V in terms of A, then find V when $A = 2$. [3 marks]