Integration techniques and applications - practice test 5

Name: _____ Date: _____

10 questions | 58 marks | 75 minutes

Reasoning and applications across exact integrals, numerical approximation, changing regions and motion.

A scientific calculator may be used.

1. For all real x, $F'(x) = \frac{2x+3}{x^2+3x+5}$ and $F(0) = 0$. [5 marks]

(a) Find F and justify that it is defined on the whole real line. [3 marks]

(b) Find $F(2) - F(0)$. [2 marks]

2. Let $I = \int_0^1 (1+x^2)^{-1}\,dx$ and $J = \int_0^1 (1+4x^2)^{-1}\,dx$. [5 marks]

(a) Evaluate I and J exactly. [3 marks]

(b) Without using decimal approximations, explain which is larger. [2 marks]

3. For $x > 0$, let $f(x) = \frac{x^2+x+1}{x(x+1)^2}$. [6 marks]

(a) Find A, B and C in $f = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$. [3 marks]

(b) Evaluate the integral of f from 1 to 2. [3 marks]

4. Let $I = \int_0^{\frac{\pi}{2}} e^x \sin x \, dx$ and $J = \int_0^{\frac{\pi}{2}} e^x \cos x \, dx$. [6 marks]

(a) Use integration by parts to show that $I + J$ equals $e^{\frac{\pi}{2}}$. [3 marks]

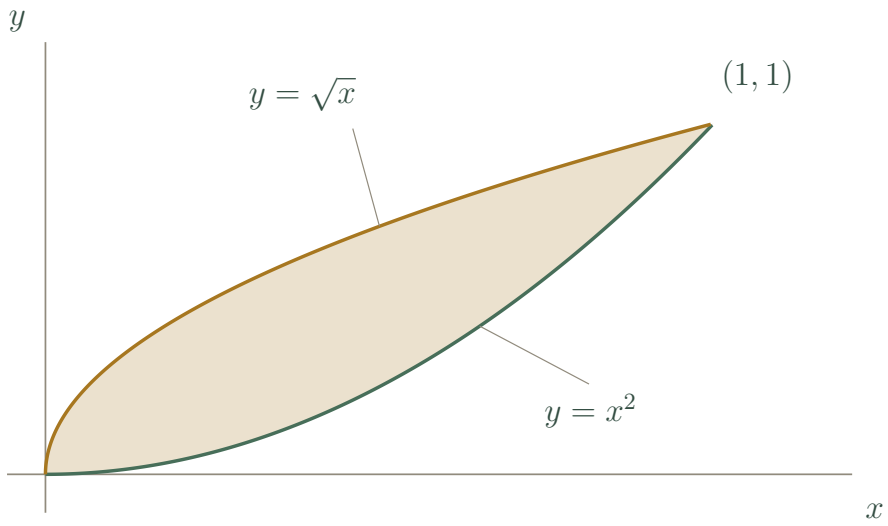
(b) Use parts on J, then solve for I. [3 marks]

5. The curve is $y = x^3 - x$ on $-2 \leq x \leq 2$. [6 marks]

(a) Find all zeros in the interval and the signed integral. [2 marks]

(b) Find the total geometric area between the curve and the axis. [4 marks]

6. A region lies between $y = \sqrt{x}$ and $y = x^2$ in the first quadrant. [9 marks]



(a) Find the intersection points and the bounded area. [3 marks]

(b) Find its volume when rotated about the x axis. [3 marks]

(c) Find its volume when rotated about the y axis, using horizontal slices. [3 marks]

7. A tank has horizontal cross-sectional area $A(h) = \pi(4 - h^2)$ m^2 at height h metres, for $0 \le h \le 2$. [5 marks]

(a) Find the volume below height h and the full capacity. [3 marks]

(b) At half the full height, what fraction of the full capacity is filled? [2 marks]

8. The table gives values of $f(x) = e^{-x^2}$. Use these values as supplied. [5 marks]

x	$f(x)$
0	1.0000
0.5	0.7788
1	0.3679

(a) Use the trapezium rule with two equal strips to estimate the integral from 0 to 1. [3 marks]

(b) A more accurate numerical value is 0.746824. Find the signed error T minus this value and the absolute error. [2 marks]

9. A periodic temperature model is $T(t) = 3 + 2\cos t + \sin^2 t$ degrees C for $0 \le t \le 2\pi$ hours. [5 marks]

(a) Find its exact average temperature over this interval. [3 marks]

(b) Explain why the average is above 3 degrees C despite the cosine term averaging to zero. [2 marks]

10. A particle moves on a straight line with velocity $v(t) = 3t^2 - 6t$ m/s for $0 \le t \le 3$. Its initial position is 2 m. [6 marks]

(a) Find its final position and average signed velocity. [3 marks]

(b) Find its total distance and average speed. [3 marks]