# Integration techniques and applications - practice test 4

Name: _____ Date: _____

10 questions | 54 marks | 75 minutes

Mixed techniques with translated inverse functions, repeated parts, horizontal regions and accumulated rates.

A scientific calculator may be used.

**1.** For all real $x$, let $f(x) = \frac{x+1}{\sqrt{x^2+2x+5}}$. [5 marks]

**(a)** Find an antiderivative using substitution. [3 marks]

**(b)** Find the member $F$ for which $F(0) = 3$. [2 marks]

**2.** Consider $f(x)=\frac{1}{\sqrt{4-(x-1)^2}}$. [5 marks]

**(a)** State its largest real open interval containing 1 and find an antiderivative there. [3 marks]

**(b)** Evaluate its integral from 1 to 2 exactly. [2 marks]

**3.** Consider $\int \frac{3x^2+1}{x^3+x+2}\,dx$ for $x \geq 0$. [5 marks]

**(a)** Find an antiderivative and justify the logarithm domain. [3 marks]

**(b)** Evaluate the definite integral from 0 to 1. [2 marks]

**4.** Let $f(x)=\frac{x^2+2x+3}{(x+1)(x+2)}$. [6 marks]

**(a)** Divide first, then give the partial fractions. [3 marks]

**(b)** Find an antiderivative and state the excluded inputs. [3 marks]

**5.** Consider $\int x^2\cos x\,dx$. [6 marks]

**(a)** Use integration by parts twice to find an antiderivative. [4 marks]

**(b)** Evaluate the integral from $0$ to $\pi$ and explain why a negative answer is possible. [2 marks]

**6.** Let $I = \int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x \, dx$. [4 marks]

**(a)** Derive a power-reduction expression for the integrand. [2 marks]

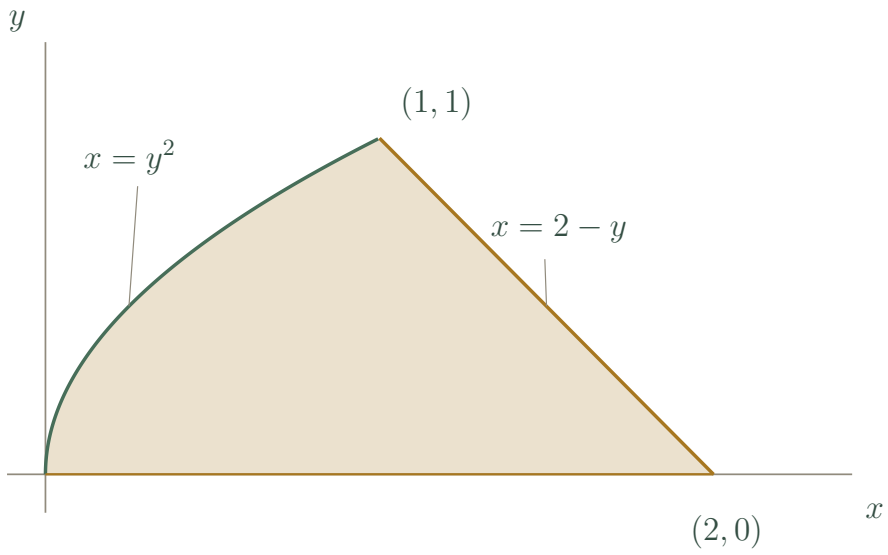
**(b)** Evaluate $I$ exactly. [2 marks]

**7.** For $x > 0$, consider $\int \ln(2x) \, dx$. [6 marks]

**(a)** Find an antiderivative using parts. [3 marks]

**(b)** Find the average value of $\ln(2x)$ on $[\frac{1}{2}, \frac{e}{2}]$. [3 marks]

**8.** A region is defined by $0 \leq y \leq 1$ and $y^2 \leq x \leq 2 - y$. [6 marks]




**(a)** Find its area using horizontal slices. [3 marks]

**(b)** Find the volume when the region is rotated about the $y$ axis. [3 marks]

**9.** Two force laws along $0 \le x \le 2$ metres are $F_1(x) = 3x + 1$ N and $F_2(x) = 3x^2$ N. [5 marks]

**(a)** Find the work done by each force over the full displacement. [3 marks]

**(b)** Does equal total work imply the forces are equal at every position? Use $x = 1$ to justify your answer. [2 marks]

**10.** A tank starts with 30 L. Water enters at $q(t) = 20te^{-t^2}$ L/min for $t \ge 0$, with no outflow. [6 marks]

**(a)** Find the amount added by time T. [3 marks]

**(b)** Find when 8 L has been added, and state the tank amount then. [3 marks]