Integration techniques and applications - practice test 3

Name: _____ Date: _____

10 questions | 44 marks | 75 minutes

Diagnose reasoning and connect integration to models.

A scientific calculator may be used.

1. Two students give $F = \frac{1}{2}\ln(x^2+3)$ and $G = \ln\sqrt{x^2+3}$ as antiderivatives of $\frac{x}{x^2+3}$. [4 marks]

(a) Show that both answers are valid for every real x. [2 marks]

(b) Explain what changes if arbitrary constants are included. [2 marks]

2. Consider $\int \frac{3e^{3x}}{1+e^{3x}}\,dx$. [4 marks]

(a) Find a suitable substitution and antiderivative. [2 marks]

(b) Evaluate it from 0 to $\frac{\ln 2}{3}$. [2 marks]

3. Let $F'(x) = \frac{4x^5+3}{x^2}$ for $x > 0$. [4 marks]

(a) Find the general antiderivative. [2 marks]

(b) Find F when $F(1) = 5$. [2 marks]

4. Consider $\int \sin^2(3x)\,dx$. [4 marks]

(a) Rewrite the integrand and integrate. [2 marks]

(b) Evaluate it from 0 to $\frac{\pi}{6}$. [2 marks]

5. For $x \geq 0$, integrate $\frac{6x+22}{(x+3)(x+4)}$. [5 marks]

(a) Find the partial fractions. [2 marks]

(b) Find the general antiderivative, then its integral from 0 to 1. [3 marks]

6. Find $\int xe^{3x}\,dx$. [5 marks]

(a) Use integration by parts, stating u and dv. [3 marks]

(b) Evaluate the integral from 0 to 1. [2 marks]

7. A proposed calculation is $\int_{-3}^{3} x^{-2}\,dx = \left[-\frac{1}{x}\right]_{-3}^{3}$. [4 marks]

(a) Explain why ordinary endpoint subtraction is invalid. [2 marks]

(b) Evaluate instead $\int_{1}^{4} x^{-2}\,dx$. [2 marks]

8. A temperature profile is $T(x) = 20 + 3\sin x$ degrees C on $0 \le x \le \pi$. [4 marks]

(a) Find its exact average value. [2 marks]

(b) Explain why the average is not 20 degrees C. [2 marks]

9. A force $F(x) = 3x^2 + 2$ N acts along displacement $0 \leq x \leq 3$ m. [4 marks]

(a) Find the work done. [2 marks]

(b) Find the average force over the displacement. [2 marks]

10. The region under $y = ax$ for $0 \leq x \leq 1$ is rotated about the x axis, with a positive. [6 marks]

(a) Express its volume in terms of a. [2 marks]

(b) Find a if the volume is 3π. [2 marks]

(c) If a doubles, explain the effect on volume. [2 marks]