Integration techniques and applications - practice test 2

Name: _____ Date: _____

10 questions | 46 marks | 75 minutes

Combine techniques and interpret geometric and practical results.

A scientific calculator may be used.

1. A model has $F'(x) = 4e^{2x}$. [4 marks]

(a) Find F if $F(0) = 5$. [2 marks]

(b) Evaluate $\int_0^{\frac{\ln(2)}{2}} 4e^{2x}\,dx$. [2 marks]

2. For $|x| < \frac{1}{2}$, consider $f(x) = \frac{2}{\sqrt{1-4x^2}}$. [4 marks]

(a) Find an antiderivative and explain the inner factor. [2 marks]

(b) Evaluate $\int_0^{\frac{1}{4}} f(x)\,dx$. [2 marks]

3. For $x > 0$, consider $\frac{2x+5}{(x+2)^2}$. [4 marks]

(a) Decompose it using both powers of the repeated factor. [2 marks]

(b) Integrate and verify by differentiation. [2 marks]

4. For $x > 0$, consider $\int x^1 \ln x \, dx$. [5 marks]

(a) Find the general antiderivative. [3 marks]

(b) Evaluate it from $x = 1$ to $x = e$. [2 marks]

5. Consider $\int \sin^3(x)\,dx$. [5 marks]

(a) Find an antiderivative using an identity and substitution. [3 marks]

(b) Evaluate it from 0 to $\frac{\pi}{2}$. [2 marks]

6. Consider $\int x\ln(x^2+1)\,dx$. [6 marks]

(a) Use substitution followed by parts to integrate. [4 marks]

(b) Evaluate the original integral from 0 to 1. [2 marks]

7. A region is bounded by $y = 3x - x^2$ and the x axis. [5 marks]

(a) Find its horizontal endpoints. [2 marks]

(b) Find the exact region area. [3 marks]

8. A region is $0 \leq y \leq 2$ and $0 \leq x \leq y^2$. [5 marks]

(a) Find its plane area. [2 marks]

(b) Find its volume about the y axis. [3 marks]

9. A tank starts with 20 L. Water enters at $q(t) = 2t + 2$ L/min for $0 \le t \le 3$. [4 marks]

(a) Find the added water and the final amount. [2 marks]

(b) Find the average inflow rate. [2 marks]

10. A student claims $G(x) = xe^{2x}$ is an antiderivative of e^{2x}. [4 marks]

(a) Differentiate G and identify the mistake. [2 marks]

(b) Give a correct antiderivative of xe^{2x}. [2 marks]