Integration techniques and applications - practice test 1

Name: _____ Date: _____

10 questions | 44 marks | 75 minutes

Select methods and establish reliable exact working.

A scientific calculator may be used.

1. Let $F'(x) = 2x + \frac{3}{x^2}$ for $x > 0$. [4 marks]

(a) Find the general antiderivative. [2 marks]

(b) Find F when $F(1) = 5$. [2 marks]

2. Consider $f(x) = \frac{1}{4 + x^2}$. [4 marks]

(a) Find an antiderivative. [2 marks]

(b) Find the exact integral from $x = 0$ to $x = 2$. [2 marks]

3. Consider $\int x(1+x^2)^2\,dx$. [5 marks]

(a) Use a substitution to find the general antiderivative. [3 marks]

(b) Evaluate the integral from 0 to 1 using transformed limits. [2 marks]

4. Consider $\int \sin^2(x)\,dx$. [4 marks]

(a) Rewrite the integrand and integrate. [2 marks]

(b) Evaluate it from 0 to $\frac{\pi}{2}$. [2 marks]

5. For $x \geq 0$, integrate $\frac{5x+13}{(x+2)(x+3)}$. [5 marks]

(a) Find the partial fractions. [2 marks]

(b) Find the general antiderivative, then its integral from 0 to 1. [3 marks]

6. Find $\int xe^x \, dx$. [5 marks]

(a) Use integration by parts, stating u and dv. [3 marks]

(b) Evaluate the integral from 0 to 1. [2 marks]

7. Consider $\int \frac{1}{x^2+2x+5}\,dx$. [4 marks]

(a) Complete the square and find an antiderivative. [2 marks]

(b) Evaluate it from $x=-1$ to $x=1$. [2 marks]

8. The curve is $y=x(x-2)$ on $0\le x\le 2$. [4 marks]

(a) Evaluate its signed integral over the interval. [2 marks]

(b) Find the geometric area and explain the sign difference. [2 marks]

9. Consider $y = 2x$ and $y = x^2$ in the first quadrant. [4 marks]

(a) Find both intersections and identify the upper curve between them. [2 marks]

(b) Find the bounded area. [2 marks]

10. The triangular region $0 \leq x \leq 2$, $0 \leq y \leq x$ is rotated. [5 marks]

(a) Find its volume about the x axis. [2 marks]

(b) Find its volume about the y axis using horizontal washers. [3 marks]