Interpreting a result in context

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Water enters at $2t+3$ litres/minute for $0 \le t \le 4$. Initially there are 10 litres. Find the final amount. [2 marks]

The added amount is $[t^2+3t]_0^4 = 28$.

Add the initial 10 litres.

Answer: 38 litres

2. Find the average temperature $20+2t$ degrees C over $0 \le t \le 4$ hours. [2 marks]

Integrate to obtain 96 degree-hours.

Divide by the 4-hour interval.

Answer: 24°C.

3. Velocity is $v = t-2$ m/s for $0 \le t \le 4$. Find displacement and total distance. [2 marks]

The signed integral cancels.

Split at $t=2$ and integrate speed on each side.

Answer: Displacement 0 m; distance 4 m.

4. A population increases at $100e^{0.1t}$ people/year for $0 \le t \le 10$. Find the increase. [2 marks]

An antiderivative is $1000e^{0.1t}$.

Subtract its $t = 0$ value from its $t = 10$ value.

Answer: $1000(e - 1)$ people, approximately 1718.

5. Marginal cost is $6q + 10$ dollars/unit. Find the cost increase when output rises from $q = 0$ to 5. [2 marks]

Integrate marginal cost over output.

$[3q^2 + 10q]_0^5 = 125.$

Answer: $125.$

6. A solid has cross-section area $\pi(1 + x)^2$ for $0 \le x \le 2$ metres. Find its volume. [2 marks]

Integrate π times $(1 + 2x + x^2)$.

Evaluate to π times $(2 + 4 + \frac{8}{3})$.

Answer: $\frac{26\pi}{3}$ m^3.