Volumes of revolution about either coordinate axis

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Rotate $0 \leq x \leq 1$, $0 \leq y \leq x^2$ about the x axis. [2 marks]

The radius squared is x^4.

Integrate π times x^4 from 0 to 1.

Answer: $\frac{\pi}{5}$ cubic units

2. Rotate $0 \leq x \leq \sqrt{2}$, $x^2 \leq y \leq 2$ about the x axis. [2 marks]

Use washer radii 2 and x^2.

Integrate $\pi(4 - x^4)$ to $\sqrt{2}$.

Answer: $\frac{16\sqrt{2}\pi}{5}$ cubic units

3. Rotate $0 \leq y \leq 3$, $0 \leq x \leq y$ about the y axis. [2 marks]

The radius is y.

$\pi \int_0^3 y^2 \, dy = 9\pi$.

Answer: 9π cubic units

4. Rotate $1 \le x \le 2$, $1 \le y \le x$ about the x axis. [2 marks]

Use outer radius x and inner radius 1.

Integrate $\pi(x^2 - 1)$ from 1 to 2.

Answer: $\frac{4\pi}{3}$ cubic units

5. Rotate the rectangle $1 \le x \le 2$, $0 \le y \le 3$ about the y axis. [2 marks]

The annular cross-section area is $\pi(4 - 1) = 3\pi$.

Multiply by height 3.

Answer: 9π cubic units

6. The region under $y = kx$ from $x = 0$ to 1 is rotated about the x axis, with $k > 0$. If volume is 12π, find k. [2 marks]

The volume is $\frac{\pi k^2}{3}$.

Solve $k^2 = 36$ and use $k > 0$.

Answer: $k = 6$.