Definite integration

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Evaluate $\int_1^e \frac{1}{x}\,dx$. [2 marks]

Use $\ln x$ on the positive interval.

Subtract $\ln 1$ from $\ln e$.

Answer: 1.

2. Evaluate $\int_0^{\pi} \sin x\,dx$. [2 marks]

Use antiderivative $-\cos x$.

The endpoint difference is 1-(-1).

Answer: 2.

3. Evaluate $\int_1^2 (2x+1)\,dx$. [2 marks]

Use x^2+x.

Subtract 2 from 6.

Answer: 4.

4. If $\int_2^5 f(x)\,dx = 7$, find $\int_5^2 f(x)\,dx$. [2 marks]

Reversing limits changes the sign.

Answer: -7.

5. Explain why ordinary endpoint subtraction is invalid for $\int_{-1}^1 x^{-2}\,dx$. [2 marks]

The continuity condition fails.

Treating the two endpoints alone would overlook the singularity.

Answer: The integrand is undefined at $x=0$ inside the interval.

6. Evaluate $\int_{-1}^1 (x^2+2)\,dx$. [2 marks]

The integrand is even, so double the integral from 0 to 1.

$2[\frac{1}{3}+2] = \frac{14}{3}$.

Answer: $\frac{14}{3}$.