Selecting/combining methods

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Integrate $\frac{\ln x}{x}$ for $x > 0$. [2 marks]

Set $u = \ln x$.

Integrate $u\,du$.

Answer: $\frac{(\ln x)^2}{2} + C$.

2. Integrate $\frac{x}{\sqrt{1-x^2}}$ on $-1 < x < 1$. [2 marks]

Use $u = 1 - x^2$.

Then $x\,dx = -\frac{du}{2}$.

Answer: $-\sqrt{1-x^2} + C$.

3. Integrate xe^{2x}. [2 marks]

Use parts with $u = x$ and $v = \frac{e^{2x}}{2}$.

Subtract the remaining exponential integral.

Answer: $e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right) + C$.

4. Integrate $\cos^2 x \sin x$. [2 marks]

Use $u = \cos x$.

The negative differential changes the sign.

Answer: $-\frac{\cos^3 x}{3} + C$.

5. Explain whether xe^{x^2} and xe^x should use the same first method. [2 marks]

The derivative of x^2 supplies the first factor x.

Differentiating x simplifies the second product without such a match.

Answer: No. Substitution is efficient for the first; parts is efficient for the second.

6. Verify that $e^x(x^2 - 2x + 2)$ integrates $x^2 e^x$. [2 marks]

Differentiate by the product rule.

The bracket sum simplifies to x^2.

Answer: It does.