Integration by parts

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\int x\cos x\,dx$. [2 marks]

Use $u = x$, $v = \sin x$.

Subtract the integral of $\sin x$.

Answer: $x\sin x + \cos x + C$.

2. Find $\int x\sin 2x\,dx$. [2 marks]

Use $v = -\frac{\cos 2x}{2}$.

Integrate the remaining cosine term.

Answer: $-\frac{x\cos 2x}{2} + \frac{\sin 2x}{4} + C$.

3. Find $\int x^2 e^x\,dx$. [2 marks]

Apply parts with $u = x^2$.

Apply parts again to the remaining xe^x term.

Answer: $e^x(x^2 - 2x + 2) + C$.

4. Find $\int x\ln x\,dx$ for $x>0$. [2 marks]

Use $u=\ln x$, $dv=x\,dx$.

The remaining integrand is $\frac{x}{2}$.

Answer: $\frac{x^2}{2}\ln x-\frac{x^2}{4}+C$.

5. Evaluate $\int_0^1 xe^x\,dx$. [2 marks]

Use antiderivative $(x-1)e^x$.

The endpoint values are 0 and -1.

Answer: 1.

6. Find $\int\arctan x\,dx$. [2 marks]

Use $u=\arctan x$, $dv=dx$.

The remaining integral is $\frac{x}{1+x^2}$.

Answer: $x\arctan x-\frac{1}{2}\ln(1+x^2)+C$.