Simple partial fractions

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Integrate $\frac{1}{x(x+1)}$. [2 marks]

Use the difference of reciprocal terms.

Integrate each and exclude $x = 0, -1$.

Answer: $\ln|x| - \ln|x+1| + C$.

2. Integrate $\frac{2}{x^2-4}$. [2 marks]

Decompose as $\frac{1}{2(x-2)} - \frac{1}{2(x+2)}$.

Integrate away from the poles.

Answer: $\frac{1}{2}\ln|x-2| - \frac{1}{2}\ln|x+2| + C$.

3. Integrate $\frac{5x+1}{(x-1)(x+1)}$. [2 marks]

Use coefficients $A = 3, B = 2$.

Their cleared numerator is $3(x+1) + 2(x-1)$.

Answer: $3\ln|x-1| + 2\ln|x+1| + C$.

4. Integrate $\frac{2x+3}{(x+1)^2}$. [2 marks]

Write $\frac{2}{x+1}+\frac{1}{(x+1)^2}$.

Integrate by logarithm and power rules.

Answer: $2\ln|x+1|-\frac{1}{x+1}+C$.

5. Integrate $\frac{x^2+1}{x^2-1}$. [2 marks]

Divide to obtain $1+\frac{2}{x^2-1}$.

Decompose the remaining fraction.

Answer: $x+\ln|x-1|-\ln|x+1|+C$.

6. Evaluate $\int_1^2 \frac{1}{x(x+1)}\,dx$. [2 marks]

Use $\ln x-\ln(x+1)$ on this positive interval.

Endpoint subtraction gives $\ln(\frac{2}{3})-\ln(\frac{1}{2})$.

Answer: $\ln(\frac{4}{3})$.