Trigonometric identities in integration

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\int \cos^2(2x)\,dx$. [2 marks]

Use $\frac{1+\cos 4x}{2}$.

Divide the cosine antiderivative by its frequency 4.

Answer: $\frac{x}{2}+\frac{\sin 4x}{8}+C$.

2. Find $\int \sin x \cos x\,dx$. [2 marks]

Use $u=\sin x$.

Integrate $u\,du$.

Answer: $\frac{\sin^2 x}{2}+C$.

3. Find $\int \sin^2 x \cos^2 x\,dx$. [2 marks]

Rewrite as $\frac{1-\cos 4x}{8}$.

Integrate each term.

Answer: $\frac{x}{8}-\frac{\sin 4x}{32}+C$.

4. Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$. [2 marks]

Use $\frac{x}{2} - \frac{\sin 2x}{4}$.

The sine endpoint terms are zero.

Answer: $\frac{\pi}{4}$.

5. Find $\int \sin x \cos(3x) \, dx$. [2 marks]

Use $\sin x \cos 3x = \frac{\sin 4x - \sin 2x}{2}$.

Integrate both terms with their frequencies.

Answer: $-\frac{\cos 4x}{8} + \frac{\cos 2x}{4} + C$.

6. Find $\int \cos^3 x \, dx$. [2 marks]

Write $(1 - \sin^2 x)\cos x$.

Use $u = \sin x$.

Answer: $\sin x - \frac{\sin^3 x}{3} + C$.