Substitution

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\int \frac{2x}{x^2+1}\,dx$. [2 marks]

Use $u = x^2 + 1$.

The integral becomes $\int \frac{du}{u}$; u is positive.

Answer: $\ln(x^2+1) + C$.

2. Find $\int \cos x \sin^3 x\,dx$. [2 marks]

Set $u = \sin x$ and $du = \cos x\,dx$.

Integrate u^3.

Answer: $\frac{\sin^4 x}{4} + C$.

3. Find $\int \frac{x}{\sqrt{4+x^2}}\,dx$. [2 marks]

Use $u = 4 + x^2$ and $x\,dx = \frac{du}{2}$.

Integrate $\frac{u^{-\frac{1}{2}}}{2}$.

Answer: $\sqrt{4+x^2} + C$.

4. Find $\int (3x+1)^5\,dx$. [2 marks]

Use $u = 3x+1$, $dx = \frac{du}{3}$.

Integrate u^5 and divide by 3.

Answer: $\frac{(3x+1)^6}{18} + C$.

5. Find $\int \frac{e^{2x}}{1+e^{2x}}\,dx$. [2 marks]

Use $u = 1+e^{2x}$; $du = 2e^{2x}\,dx$.

Integrate $\frac{du}{2u}$.

Answer: $\frac{1}{2}\ln(1+e^{2x}) + C$.

6. Evaluate $\int_0^2 x(1+x^2)\,dx$. [2 marks]

With $u = 1+x^2$, the limits become 1 and 5.

$\frac{1}{2}\int_1^5 u\,du = \frac{25-1}{4} = 6$.

Answer: 6.