Inverse-trigonometric derivatives and integrals

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\int \frac{3}{9+x^2}\,dx$. [2 marks]

Use the arctangent rule with $a = 3$.

The factor 3 cancels the outside $\frac{1}{3}$.

Answer: $\arctan\left(\frac{x}{3}\right) + C$.

2. Find $\int \frac{2}{\sqrt{1-4x^2}}\,dx$. [2 marks]

Use $u = 2x$ and $du = 2\,dx$.

The transformed integral is $\int (1-u^2)^{-\frac{1}{2}}\,du$.

Answer: $\arcsin(2x) + C$, $|x| < \frac{1}{2}$.

3. Find $\int \frac{1}{1+9x^2}\,dx$. [2 marks]

Differentiate $\arctan(3x)$ to see the extra factor 3.

Divide by 3.

Answer: $\frac{1}{3}\arctan(3x) + C$.

4. Evaluate $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$. [2 marks]

The antiderivative is $\arcsin x$.

Evaluate $\arcsin\left(\frac{1}{2}\right) - \arcsin(0)$.

Answer: $\frac{\pi}{6}$.

5. Find $\int \frac{1}{x^2+16}\,dx$. [2 marks]

Use $a = 4$ in the arctangent pattern.

Check the chain factor.

Answer: $\frac{1}{4}\arctan\left(\frac{x}{4}\right) + C$.

6. Differentiate $\arccos\left(\frac{x}{2}\right)$ on $-2 < x < 2$. [2 marks]

Use derivative $-\frac{u'}{\sqrt{1-u^2}}$.

Simplify with $u = \frac{x}{2}$.

Answer: $-\frac{1}{\sqrt{4-x^2}}$.