Vectors in three dimensions - practice test 6

Name: _____ Date: _____

10 questions | 73 marks | 75 minutes

Interpret data, diagnose errors and solve connected spatial problems, finishing with parameter cases.

A scientific calculator may be used.

1. A drone travels in straight segments from A to B, then B to C. Coordinates are in metres. [6 marks]

Point	x (m)	y (m)	z (m)
A	0	0	0
B	3	4	0
C	3	4	12

(a) Find the total path length. [2 marks]

(b) Find the displacement from A to C and its magnitude. [2 marks]

(c) Give a unit vector in the direction from A to C. [2 marks]

2. A vector has the form $\mathbf{a} = (1, -1, 2) + \lambda(1, 1, 0)$, where λ is real, and $|\mathbf{a}| = \sqrt{14}$. [7 marks]

(a) Verify that the two displayed component directions are perpendicular. [2 marks]

(b) Find every possible value of λ. [3 marks]

(c) If $\mathbf{a} \cdot (1, 1, 0) > 0$, find a. [2 marks]

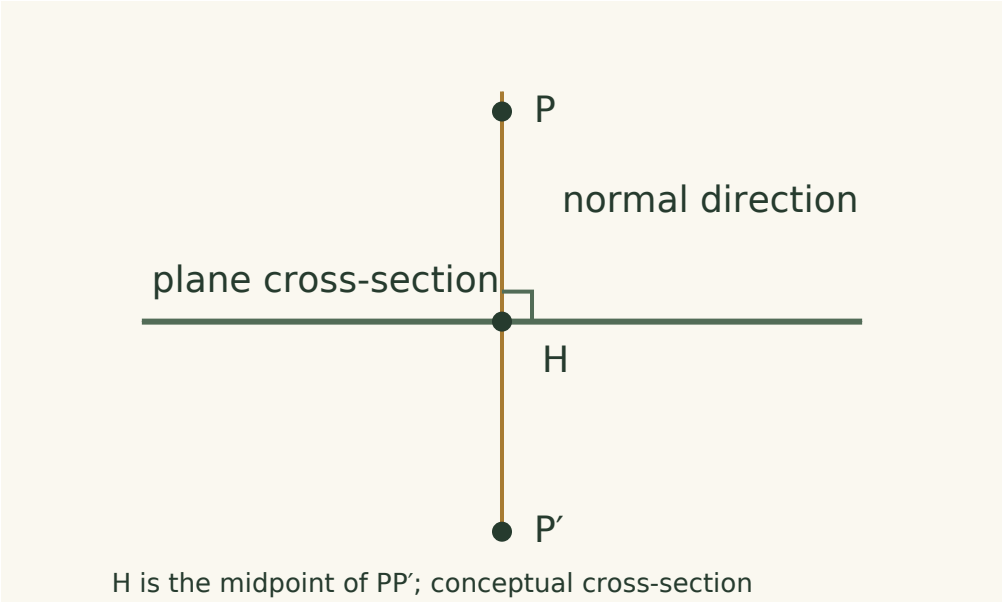
3. For $\mathbf{u} = (1, 2, -1)$ and $\mathbf{v} = (2, -1, 3)$, a student claims $\mathbf{u} \times \mathbf{v} = (5, 5, -5)$. [7 marks]

(a) Use a dot-product check to show that the claimed cross product is incorrect. [2 marks]

(b) Find the correct cross product and check both perpendicularities. [3 marks]

(c) Find the area of the triangle spanned by u and v. [2 marks]

4. Point $P=(3,0,0)$ is reflected in the plane $x-y+z=2$. The reflection P' is defined so that the perpendicular foot H is the midpoint of PP'. [7 marks]



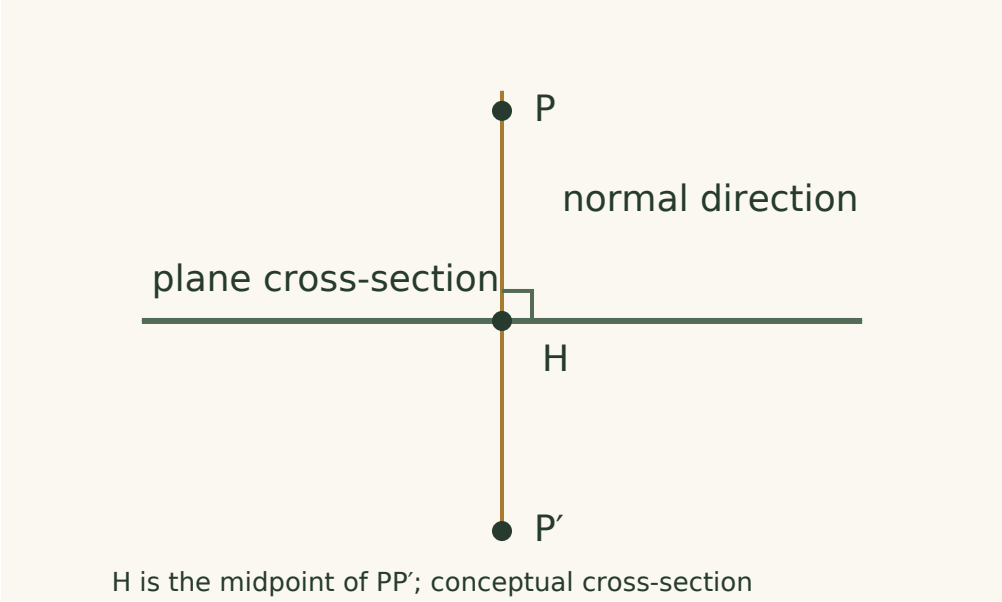
H is the midpoint of PP′; conceptual cross-section

(a) Find H. [3 marks]

(b) Find the reflection P'. [2 marks]

Question 4 (continued)

Point $P = (3, 0, 0)$ is reflected in the plane $x - y + z = 2$. The reflection P' is defined so that the perpendicular foot H is the midpoint of PP'.



H is the midpoint of PP′; conceptual cross-section

(c) Find the length PP' and explain why it is twice the distance of P from the plane.
[2 marks]

5. Two robots move in a common coordinate system according to $P(t) = (t, 0, 0)$ and $Q(t) = (2, 3 - t, 0)$ for $0 \leq t \leq 5$, with t in seconds and coordinates in metres. [7 marks]

(a) Write their squared separation at time t. [2 marks]

(b) Find their minimum separation and the time when it occurs, without calculus. [3 marks]

(c) Do the robots collide? Justify. [2 marks]

6. A straight cable segment has endpoints $A(1, 0, 0)$ and $B(3, 2, 2)$. An infinite plane has equation $x + y + z = 10$. [7 marks]

(a) Parametrise the cable segment, stating the parameter interval. [2 marks]

(b) Find where the complete line through A and B meets the plane. [2 marks]

(c) Does the cable segment meet the plane? Find its closest endpoint and that endpoint's distance from the plane. [3 marks]

7. Two planes are $\Pi_1 : x + y + z = 4$ and $\Pi_2 : 2x - y + z = 2$. [7 marks]

(a) Find a direction vector for their intersection. [2 marks]

(b) Give a vector equation for the intersection line. [3 marks]

(c) Find the smaller angle between the planes to one decimal degree. [2 marks]

8. Two support lines are $\ell_1 : (1,0,0) + s(1,1,1)$ and $\ell_2 : (0,2,0) + t(1,-1,0)$, for real s, t. [9 marks]

(a) Find a common perpendicular direction and the shortest line separation. [3 marks]

(b) Find the closest points P on ℓ_1 and Q on ℓ_2. [4 marks]

(c) Verify the joining vector is perpendicular to both lines. [2 marks]

9. Consider the homogeneous system $x+y+z=0,\ 2x-y+z=0,\ 3x+2z=0$. [7 marks]

(a) Explain why the system cannot have no solution. [2 marks]

(b) Find every solution and interpret the common geometry. [3 marks]

(c) If the extra constraint $x=0$ is imposed, what remains? [2 marks]

10. A system depends on real parameters a and b: $x+y+z=3$, $x+2y+3z=6$, $x+2y+az=b$. [9 marks]

(a) For $a \neq 3$, find the unique solution in terms of a, b. [4 marks]

(b) Classify and parametrise the solution set when $a=3, b=6$. [3 marks]

(c) Classify the case $a=3$, $b \neq 6$ and justify it. [2 marks]