Vectors in three dimensions - practice test 5

Name: _____ Date: _____

10 questions | 60 marks | 70 minutes

Use parameters, proof and perpendicularity in connected tasks.

A scientific calculator may be used.

1. A constant force $\mathbf{F} = (2, 0, 1)$ N acts during displacement $\mathbf{d} = (-2, 1, 2)$ m. [7 marks]

(a) Calculate the work done. [2 marks]

(b) Find the vector component of F along the displacement direction. [3 marks]

(c) Explain the sign of the work in relation to the projected force. [2 marks]

2. Let $\mathbf{a}, \mathbf{b}$ be real three-dimensional vectors with $\mathbf{b} \neq \mathbf{0}$. Define $\mathbf{p} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2}\mathbf{b}$ and $\mathbf{q} = \mathbf{a} - \mathbf{p}$. [7 marks]

(a) Prove that $\mathbf{q} \cdot \mathbf{b} = 0$. [3 marks]

(b) Hence prove that $|\mathbf{a}|^2 = |\mathbf{p}|^2 + |\mathbf{q}|^2$. [2 marks]

(c) For $\mathbf{a} = (5, 1, 2)$ and $\mathbf{b} = (1, 0, 1)$, calculate $\mathbf{p}$ and $\mathbf{q}$ and check the squared-length equality. [2 marks]

3. A parallelogram $ABCD$ has consecutive vertices $A(1, 0, 2)$, $B(3, 1, 1)$, $C(4, 4, 3)$. [6 marks]

(a) Find D. [2 marks]

(b) Verify that its diagonals bisect one another. [2 marks]

(c) Is this parallelogram a rhombus? Justify. [2 marks]

4. Let $\mathbf{a} = (5, 2, 3)$ and $\mathbf{b} = (1, 1, 0)$. [5 marks]

(a) Find the vector projection of a along b. [2 marks]

(b) Find and verify the perpendicular component. [3 marks]

5. Two planes have equations $\Pi_1 : x + 2y - z = 4$ and $\Pi_2 : 2x + 4y - 2z = k$. [7 marks]

(a) Explain why their normals are parallel. [2 marks]

(b) For which k are the planes coincident? Describe all remaining cases. [2 marks]

(c) For $k = 10$, find the plane separation. [3 marks]

6. Consider $\ell : \mathbf{r} = (0, 0, 5) + t(1, -1, a)$ and plane $x + y + z = 3$. [4 marks]

(a) For a nonzero, find the parameter of the intersection. [2 marks]

(b) Classify the case $a = 0$. [2 marks]

7. $P = (0, 5, 2)$ and $\ell : \mathbf{r} = (1, 0, 0) + t(1, 1, 0)$. [5 marks]

(a) Find the parameter of the closest point Q. [2 marks]

(b) Find Q and the shortest distance. [3 marks]

8. Consider lines $\ell_1 = s(1, 1, 0)$ and $\ell_2 = (0, 5, 1) + t(1, 0, 1)$. [7 marks]

(a) Show that the lines are skew. [2 marks]

(b) Find the closest points P and Q, and their distance. [5 marks]

9. An augmented system has rows $[1, 1, 1 \mid 6]$, $[2, 3, 1 \mid 13]$, $[1, 0, 2 \mid 5]$. A student applies $R_2 \leftarrow R_2 - 2R_1$ but writes $[0, 1, -1 \mid 13]$. [7 marks]

(a) Identify the error and write the correct new row 2. [2 marks]

(b) Apply $R_3 \leftarrow R_3 - R_1$ and compare the last two rows. [2 marks]

(c) Describe the entire solution set and its geometry. [3 marks]

10. An augmented system has rows $[1, 2, -1 \mid 4]$, $[3, 7, -2 \mid 13]$, $[0, 0, 1 \mid 5]$. [5 marks]

(a) Apply $R_2 \leftarrow R_2 - 3R_1$. [2 marks]

(b) Find the unique solution. [3 marks]