Vectors in three dimensions - practice test 4

Name: _____ Date: _____

10 questions | 51 marks | 70 minutes

Solve mixed geometric and algebraic problems.

A scientific calculator may be used.

1. Let $\mathbf{a} = (1, 4, 0)$ and $\mathbf{b} = (0, 1, 1)$. [5 marks]

(a) Calculate the dot product. [2 marks]

(b) Find their angle in degrees to one decimal place. [3 marks]

2. Let $\mathbf{a} = (u, 4, 2)$, with u real. [4 marks]

(a) Find u if $|\mathbf{a}| = \sqrt{29}$. [2 marks]

(b) Find u if a is perpendicular to $(4, -1, 1)$. [2 marks]

3. A triangular panel has vertices $O(0,0,0)$, $A(2,0,0)$, $B(0,k,1)$, where $k>0$. Its area is $\sqrt{10}$ square metres; coordinates are in metres. [7 marks]

(a) Find $\overrightarrow{OA}\times\overrightarrow{OB}$ in terms of k. [2 marks]

(b) Use the area to find k. [3 marks]

(c) Find a unit normal with positive z component. [2 marks]

4. A cell is spanned by $\mathbf{a} = (1, 1, 0)$, $\mathbf{b} = (0, 2, 1)$, $\mathbf{c} = (4, 0, 3)$. [4 marks]

(a) Find the scalar triple product. [2 marks]

(b) Find its volume and say whether the vectors are coplanar. [2 marks]

5. Planes $\Pi_1 : x + y = 4$ and $\Pi_2 : z = 2$ are given. [6 marks]

(a) Find a direction for their intersection line. [2 marks]

(b) Give a vector equation for the intersection. [2 marks]

(c) Find the smaller angle between the planes. [2 marks]

6. Consider the family of lines $\ell_k : \mathbf{r} = (1, 0, 2) + t(1, 2, k)$ and plane $\Pi : x + 2y + 2z = 5$. [6 marks]

(a) Find k for which the line direction is parallel to the plane. [2 marks]

(b) For this value of k, decide whether the line is disjoint from the plane or contained in it. [2 marks]

(c) Find k for which the line is perpendicular to the plane. [2 marks]

7. Lines $\ell_1 = (s, 0, 0)$ and $\ell_2 = (t, 4, 4)$ model two straight rails. [4 marks]

(a) Explain why the cross-product skew-distance formula is unsuitable. [2 marks]

(b) Find the rail separation. [2 marks]

8. $P = (4, 2, 3)$ and plane $\Pi : x + y + z = 0$. [4 marks]

(a) Find the perpendicular foot Q on the plane. [2 marks]

(b) Find PQ and verify perpendicularity. [2 marks]

9. Consider $x + y + z = 4$, $2x + 2y + 2z = 8$, and $x - y = 0$. [4 marks]

(a) Describe every solution. [2 marks]

(b) If the constant 8 in equation 2 is increased by 1, how does the solution set change? [2 marks]

10. Three unknown steady flow rates x, y and z, in litres per second, satisfy $x+y=30$, $y+z=25$, $x+z=35$. [7 marks]

(a) Write an augmented matrix for the system. [2 marks]

(b) Solve for the three rates and check each original balance. [3 marks]

(c) Explain what a negative solution would mean for the assumed flow directions. [2 marks]