Vectors in three dimensions - practice test 3

Name: _____ Date: _____

10 questions | 58 marks | 70 minutes

Apply spatial methods and justify each conclusion.

A scientific calculator may be used.

1. A triangular frame has vertices $A(0,0,0)$, $B(6,0,0)$, $C(0,3,3)$. Its centroid G is defined to have position vector $\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$. [6 marks]

(a) Find the midpoint M of BC. [2 marks]

(b) Find G and show that it lies on the median AM. [2 marks]

(c) State $AG:GM$ and explain why this is internal division. [2 marks]

2. A survey displacement is $\mathbf{a} = (3, 2, 2)$ metres. [4 marks]

(a) Find its exact length. [2 marks]

(b) Find a length-3 vector in the opposite direction. [2 marks]

3. Let $\mathbf{a} = (3, 2, 3)$ and $\mathbf{b} = (1, 1, 0)$. [5 marks]

(a) Find the vector projection of a along b. [2 marks]

(b) Find and verify the perpendicular component. [3 marks]

4. For arbitrary real vectors a and b, consider $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})$. [7 marks]

(a) Expand and simplify this expression. [3 marks]

(b) Explain why the diagonals of a rhombus are perpendicular. [2 marks]

(c) Check the identity for $a=(3,1,2)$, $b=(1,0,1)$. [2 marks]

5. $A=(1,0,3)$, $B=(2,1,3)$ and $C=(1,1,4)$ are three points. [6 marks]

(a) Find a nonzero normal to their plane. [2 marks]

(b) Write a Cartesian plane equation. [2 marks]

(c) Check whether $P=(2,0,3)$ lies in the plane. [2 marks]

6. Two straight tracks are modelled by $\mathbf{r} = (1, 0, 2) + t(1, 2, -1)$ and $\mathbf{r} = (3, 1, 0) + s(-1, 1, 1)$, with real parameters. [7 marks]

(a) Find the intersection, using a separate parameter for each track. [3 marks]

(b) If t and s are each elapsed seconds after the same start time, do two objects following these equations collide? [2 marks]

(c) Explain why intersecting tracks alone would not guarantee a collision. [2 marks]

7. Consider $\ell : \mathbf{r} = (0, 0, 3) + t(1, -1, a)$ and plane $x + y + z = 3$. [4 marks]

(a) For a nonzero, find the parameter of the intersection. [2 marks]

(b) Classify the case $a = 0$. [2 marks]

8. $P = (0, 3, 2)$ and $\ell : \mathbf{r} = (1, 0, 0) + t(1, 1, 0)$. [5 marks]

(a) Find the parameter of the closest point Q. [2 marks]

(b) Find Q and the shortest distance. [3 marks]

9. Three planes have equations $x+y+z=1$, $x-y+z=3$ and $2x+2z=k$, where k is real. [7 marks]

(a) Eliminate y from the first two equations. What value of k allows a common solution? [2 marks]

(b) For that value of k, describe the entire common solution set. [3 marks]

(c) Describe the geometry when k is any other value. [2 marks]

10. Consider lines $\ell_1 = s(1, 1, 0)$ and $\ell_2 = (0, 3, 1) + t(1, 0, 1)$. [7 marks]

(a) Show that the lines are skew. [2 marks]

(b) Find the closest points P and Q, and their distance. [5 marks]