Vectors in three dimensions - practice test 2

Name: _____ Date: _____

10 questions | 48 marks | 70 minutes

Connect algebra with geometric interpretation.

A scientific calculator may be used.

1. Let $\mathbf{a} = (u, 2, 2)$, with u real. [4 marks]

(a) Find u if $|\mathbf{a}| = \sqrt{17}$. [2 marks]

(b) Find u if a is perpendicular to $(2, -1, 1)$. [2 marks]

2. A constant force $\mathbf{F} = (6, 2, -1)$ N moves a component through displacement $(3, -1, 2)$ m. [4 marks]

(a) Find the work done. [2 marks]

(b) Resolve the force into a component along the x axis and a perpendicular component. [2 marks]

3. A cell is spanned by $\mathbf{a} = (1, 1, 0)$, $\mathbf{b} = (0, 2, 1)$, $\mathbf{c} = (2, 0, 3)$. [4 marks]

(a) Find the scalar triple product. [2 marks]

(b) Find its volume and say whether the vectors are coplanar. [2 marks]

4. Two sides of a triangular panel are $(2, 1, 0)$ m and $(0, 1, 2)$ m. [6 marks]

(a) Find their cross product. [2 marks]

(b) Find the panel area. [2 marks]

(c) Give a unit normal with positive x component. [2 marks]

5. Planes $\Pi_1 : x + y = 2$ and $\Pi_2 : z = 2$ are given. [6 marks]

(a) Find a direction for their intersection line. [2 marks]

(b) Give a vector equation for the intersection. [2 marks]

(c) Find the smaller angle between the planes. [2 marks]

6. Consider lines $\ell_1 = s(1, 1, 0)$ and $\ell_2 = (0, 2, 1) + t(1, 0, 1)$. [7 marks]

(a) Show that the lines are skew. [2 marks]

(b) Find the closest points P and Q, and their distance. [5 marks]

7. Lines $\ell_1 = (s, 0, 0)$ and $\ell_2 = (t, 2, 4)$ model two straight rails. [4 marks]

(a) Explain why the cross-product skew-distance formula is unsuitable. [2 marks]

(b) Find the rail separation. [2 marks]

8. $P = (2, 2, 3)$ and plane $\Pi : x + y + z = 0$. [4 marks]

(a) Find the perpendicular foot Q on the plane. [2 marks]

(b) Find PQ and verify perpendicularity. [2 marks]

9. Consider $x+y+z=2$, $2x+2y+2z=4$, and $x-y=0$. [4 marks]

(a) Describe every solution. [2 marks]

(b) If the constant 4 in equation 2 is increased by 1, how does the solution set change? [2 marks]

10. An augmented system has rows $[1, 2, -1 \mid 4]$, $[3, 7, -2 \mid 13]$, $[0, 0, 1 \mid 2]$. [5 marks]

(a) Apply $R_2 \leftarrow R_2 - 3R_1$. [2 marks]

(b) Find the unique solution. [3 marks]