Vectors in three dimensions - practice test 1

Name: _____ Date: _____

10 questions | 51 marks | 70 minutes

Build confident vector and coordinate methods.

A scientific calculator may be used.

1. A survey displacement is $\mathbf{a} = (1, 2, 2)$ metres. [4 marks]

(a) Find its exact length. [2 marks]

(b) Find a length-3 vector in the opposite direction. [2 marks]

2. A has position $(1, 1, 0)$ and B has position $(7, 4, 9)$. [4 marks]

(a) Find P with $AP : PB = 1 : 2$ internally. [2 marks]

(b) Find the midpoint of PB. [2 marks]

3. Let $\mathbf{a} = (1, 1, 0)$ and $\mathbf{b} = (0, 1, 1)$. [5 marks]

(a) Calculate the dot product. [2 marks]

(b) Find their angle in degrees to one decimal place. [3 marks]

4. Let $\mathbf{a} = (1, 2, 3)$ and $\mathbf{b} = (1, 1, 0)$. [5 marks]

(a) Find the vector projection of a along b. [2 marks]

(b) Find and verify the perpendicular component. [3 marks]

5. Two sides of a triangular panel are $(1, 1, 0)$ m and $(0, 1, 2)$ m. [6 marks]

(a) Find their cross product. [2 marks]

(b) Find the panel area. [2 marks]

(c) Give a unit normal with positive x component. [2 marks]

6. A line is $\mathbf{r} = (1, 1, 2) + t(2, -1, 3)$. [6 marks]

(a) Find the point at $t = 2$. [2 marks]

(b) Is $(5, -1, 9)$ on the line? Justify. [2 marks]

(c) State a Cartesian form. [2 marks]

7. $A=(1,0,1)$, $B=(2,1,1)$ and $C=(1,1,2)$ are three points. [6 marks]

(a) Find a nonzero normal to their plane. [2 marks]

(b) Write a Cartesian plane equation. [2 marks]

(c) Check whether $P=(2,0,1)$ lies in the plane. [2 marks]

8. A line is $\mathbf{r}=(0,1,1)+t(1,1,1)$ and a plane is $x+y+z=8$. [4 marks]

(a) Find their intersection. [2 marks]

(b) Find the smaller angle between the line and plane, to one decimal degree. [2 marks]

9. $P = (0, 1, 2)$ and $\ell : \mathbf{r} = (1, 0, 0) + t(1, 1, 0)$. [5 marks]

(a) Find the parameter of the closest point Q. [2 marks]

(b) Find Q and the shortest distance. [3 marks]

10. Solve the system $x + y + z = 4$, $2x - y + z = 1$, $x + 2y - z = 4$. [6 marks]

(a) Eliminate x to obtain two equations in y, z. [3 marks]

(b) Solve and check the original system. [3 marks]