Vectors in three dimensions - practice test 1

10 questions | 51 marks | 70 minutes

Build confident vector and coordinate methods.

Equivalent correct methods earn the corresponding marks.

1. A survey displacement is $\mathbf{a} = (1, 2, 2)$ metres. [4 marks]

(a) Find its exact length. [2 marks]

The sum of squared components is 9.

Take its positive square root.

Answer: 3 m

(b) Find a length-3 vector in the opposite direction. [2 marks]

The unit direction is $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$.

Multiply this by -3.

Answer: $(-1,\ -2,\ -2)$

2. A has position $(1, 1, 0)$ and B has position $(7, 4, 9)$. [4 marks]

(a) Find P with $AP : PB = 1 : 2$ internally. [2 marks]

Use $\frac{2\mathbf{a} + \mathbf{b}}{3}$.

Substitution gives $(3, 2, 3)$.

Answer: $(3, 2, 3)$

(b) Find the midpoint of PB. [2 marks]

Average P and B component by component.

The result is $(5, 3, 6)$.

Answer: $(5, 3, 6)$

3. Let $\mathbf{a} = (1, 1, 0)$ and $\mathbf{b} = (0, 1, 1)$. [5 marks]

(a) Calculate the dot product. [2 marks]

Add the products of matching components: 1.

Answer: 1

(b) Find their angle in degrees to one decimal place. [3 marks]

$\cos\theta = \frac{1}{2}$.

Use inverse cosine in degree mode.

Answer: 60.0°

4. Let $\mathbf{a} = (1, 2, 3)$ and $\mathbf{b} = (1, 1, 0)$. [5 marks]

(a) Find the vector projection of a along b. [2 marks]

Projection coefficient is $\frac{3}{2}$.

Multiply this scalar by b.

Answer: $(\frac{3}{2}, \frac{3}{2}, 0)$

(b) Find and verify the perpendicular component. [3 marks]

Subtract the projection from a. Dotting $(-\frac{1}{2}, \frac{1}{2}, 3)$ with b gives 0.

Answer: $(-\frac{1}{2}, \frac{1}{2}, 3)$

5. Two sides of a triangular panel are $(1, 1, 0)$ m and $(0, 1, 2)$ m. [6 marks]

(a) Find their cross product. [2 marks]

Use the determinant/component formula.

The result is $(2, -2, 1)$.

Answer: $(2, -2, 1)$

(b) Find the panel area. [2 marks]

The parallelogram area is 3.

Halve it for the triangle.

Answer: $\frac{3}{2}$ m^2

(c) Give a unit normal with positive x component. [2 marks]

The cross product has positive x component.

Divide it by its magnitude.

Answer: $(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3})$

6. A line is $\mathbf{r} = (1, 1, 2) + t(2, -1, 3)$. [6 marks]

(a) Find the point at $t = 2$. [2 marks]

Substitute the same parameter in all components.

Answer: $(5, -1, 8)$

(b) Is $(5, -1, 9)$ on the line? Justify. [2 marks]

The x and y coordinates require $t = 2$.

At $t = 2$ the z coordinate is 8, not 9.

Answer: No.

(c) State a Cartesian form. [2 marks]

Eliminate the common parameter.

Answer: $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z-2}{3}$.

7. $A=(1,0,1)$, $B=(2,1,1)$ and $C=(1,1,2)$ are three points. [6 marks]

(a) Find a nonzero normal to their plane. [2 marks]

Use $AB\times AC$.

The side directions are $(1,1,0)$ and $(0,1,1)$.

Answer: $(1,-1,1)$

(b) Write a Cartesian plane equation. [2 marks]

Use the normal $(1,-1,1)$.

The constant is $n\cdot A=2$.

Answer: $x-y+z=2$

(c) Check whether $P=(2,0,1)$ lies in the plane. [2 marks]

Increasing x by 1 increases the left side by 1.

The plane constant is unchanged.

Answer: No.

8. A line is $\mathbf{r}=(0,1,1)+t(1,1,1)$ and a plane is $x+y+z=8$. [4 marks]

(a) Find their intersection. [2 marks]

Substitution gives $2+3t=8$.

Thus $t=2$; substitute into the line.

Answer: $(2,3,3)$

(b) Find the smaller angle between the line and plane, to one decimal degree. [2 marks]

The line direction equals the plane normal.

The line is perpendicular to the plane.

Answer: 90.0°

9. $P=(0,1,2)$ and $\ell:\mathbf{r}=(1,0,0)+t(1,1,0)$. [5 marks]

(a) Find the parameter of the closest point Q. [2 marks]

Perpendicularity gives $(1+t,t-1,-2)\cdot(1,1,0)=0$.

This simplifies to $2t+1-1=0$.

Answer: $t=0$

(b) Find Q and the shortest distance. [3 marks]

Substitute the parameter into the line.

Calculate $|Q-P|=\sqrt{6}$.

Answer: $Q=(1,0,0)$, distance $\sqrt{6}$.

10. Solve the system $x+y+z=4$, $2x-y+z=1$, $x+2y-z=4$. [6 marks]

(a) Eliminate x to obtain two equations in y,z. [3 marks]

Subtract twice equation 1 from equation 2.

Subtract equation 1 from equation 3.

Answer: $-3y-z=-7$ and $y-2z=0$.

(b) Solve and check the original system. [3 marks]

The two-variable equations give $y=2,z=1$; then $x=1$.

Substitution satisfies all three originals.

Answer: $(x,y,z)=(1,2,1)$