Row operations and geometric interpretation of solution sets

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Apply $R_2 \leftarrow R_2 - 3R_1$ to $[1, 2, -1 \mid 4]$ and $[3, 7, -2 \mid 13]$. [2 marks]

Subtract $[3, 6, -3 \mid 12]$ entry by entry.

The new equation is $y + z = 1$.

Answer: $[0, 1, 1 \mid 1]$

2. Interpret rows $[1, 0, 0 \mid 2]$, $[0, 1, 0 \mid -1]$, $[0, 0, 1 \mid 3]$. [2 marks]

Every variable has its own pivot.

Read the augmented constants.

Answer: Unique solution $(2, -1, 3)$.

3. Interpret rows $[1, 2, 0 \mid 5]$, $[0, 0, 1 \mid 1]$, $[0, 0, 0 \mid 0]$. [2 marks]

Choose $y = t$.

Then $x = 5 - 2t$ and $z = 1$.

Answer: Line $(5 - 2t, t, 1)$.

4. Interpret a consistent three-variable system with one pivot. [2 marks]

One independent equation restricts three variables.

Two independent directions remain free.

Answer: A plane with two free parameters.

5. Is multiplying a row by zero an equivalence operation? [2 marks]

It destroys the row's constraint.

It cannot be reversed.

Answer: No.

6. Check $(2, 1, 0)$ in $x + y + z = 3$, $2x - y + z = 3$, $x - y - z = 1$. [2 marks]

The three left sides are 3, 3 and 1.

Each agrees with its original constant.

Answer: It is a correct solution.