Systems of three linear equations

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $x+y+z=9$, $x-y=1$, $z=2$. [2 marks]

Write $x=y+1$.

Then $2y+3=9$, so $y=3$.

Answer: $(4, 3, 2)$

2. Solve $x+y=3$, $y+z=5$, $x+z=4$. [2 marks]

Add the first two and subtract the third: $2y=4$.

Back-substitute.

Answer: $(1, 2, 3)$

3. Classify $x+y+z=2$, $2x+2y+2z=5$, $z=1$. [2 marks]

Twice the first equation would require left side 4 in the second.

The second constant is 5, giving a contradiction.

Answer: No solution.

4. Parametrise $x+y=2$, $z=1$, $2x+2y=4$. [2 marks]

The last equation is redundant.

Choose $y=t$ and solve for x.

Answer: $(2-t, t, 1)$, real t.

5. Solve $x+2y+z=8$, $y-z=1$, $x-z=2$. [2 marks]

Use $y=z+1$ and $x=z+2$.

Substitution gives $4z+4=8$.

Answer: $(3, 2, 1)$

6. What does a unique solution of three independent plane equations represent? [2 marks]

Each equation imposes an independent restriction.

No free direction remains.

Answer: Exactly one common point of all three planes.