# Closest points and distances including skew lines

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

**1.** Find the closest point on $(t, 0, 0)$ to $(3, 4, 5)$ and its distance. [2 marks]

Perpendicularity requires $t = 3$.

The remaining displacement is $(0, 4, 5)$.

**Answer:** $Q = (3, 0, 0)$, distance $\sqrt{41}$.

**2.** Find the distance from $(1, 2, 3)$ to $x + y + z = 0$. [2 marks]

Use $\frac{|6|}{\sqrt{3}}$.

Simplify to $2\sqrt{3}$.

**Answer:** $2\sqrt{3}$

**3.** Find the foot of the perpendicular from $(1, 2, 3)$ to $x + y + z = 0$. [2 marks]

Subtract $[\frac{6}{3}](1, 1, 1)$ from the point.

The result lies in the plane.

**Answer:** $(-1, 0, 1)$

**4.** Find the closest points and distance between $\ell_1 = s(1, 1, 0)$ and $\ell_2 = (0, 2, 1) + t(1, 0, 1)$. [4 marks]

The joining vector is $(t - s, 2 - s, 1 + t)$. Perpendicularity gives $t + 2 - 2s = 0$ and $2t - s + 1 = 0$.

Solve to obtain $s = 1, t = 0$. The joining vector is $(-1, 1, 1)$, which has length $\sqrt{3}$.

**Answer:** $P = (1, 1, 0)$ and $Q = (0, 2, 1)$; distance $\sqrt{3}$.

**5.** Find the distance between $(s, 0, 0)$ and $(t, 3, 4)$. [2 marks]

The lines are parallel to the $x$ axis.

The perpendicular displacement is $(0, 3, 4)$.

**Answer:** $5$

**6.** For line $(1, 0, 0) + t(1, 1, 0)$ and $P(0, 1, 2)$, find the closest $t$ and distance. [2 marks]

The dot product $(1 + t, t - 1, -2) \cdot (1, 1, 0) = 2t$ vanishes at 0.

Then the squared distance is $1 + 1 + 4 = 6$.

**Answer:** $t = 0$, distance $\sqrt{6}$.