Geometric areas and perpendicular vectors

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find the parallelogram area spanned by $(1, 0, 0)$ and $(0, 2, 2)$. [2 marks]

The cross product is $(0, -2, 2)$.

Its magnitude is $\sqrt{8}$.

Answer: $2\sqrt{2}$ square units

2. Find triangle area for $A(1, 0, 0)$, $B(3, 0, 0)$, $C(1, 4, 0)$. [2 marks]

The perpendicular sides have lengths 2 and 4.

Use half their product.

Answer: 4 square units

3. Give a length-5 vector perpendicular to $(1, 0, 0)$ and $(0, 1, 0)$. [2 marks]

The cross-product direction is the z axis.

Scale its unit direction by 5.

Answer: $(0, 0, 5)$ or $(0, 0, -5)$

4. Find triangle area for sides $(1, 2, 0)$ and $(0, 1, 3)$. [2 marks]

Their cross product is $(6, -3, 1)$.

Its squared magnitude is 46; halve the magnitude.

Answer: $\frac{\sqrt{46}}{2}$ square units

5. Explain why replacing b by $3b$ triples the spanned parallelogram area. [2 marks]

Use linearity of the cross product.

Use $|3\mathbf{v}| = 3|\mathbf{v}|$.

Answer: The cross product becomes $3(\mathbf{a} \times \mathbf{b})$, so its magnitude triples.

6. Find a unit normal to sides $(2, 0, 0)$, $(0, 3, 4)$ with positive z component. [2 marks]

The cross product is $(0, -8, 6)$ with length 10.

Normalise using the positive-z orientation.

Answer: $(0, -\frac{4}{5}, \frac{3}{5})$