Functions and graph sketching - Practice test 6

Name: _____ Date: _____

10 questions | 50 marks | 75 minutes

Ten multipart questions combining exact calculations, domains, graph evidence, interpretation and reasoning. Show working and label key graph features.

A scientific calculator may be used.

1. Let $f(x)=\sqrt{1-x}$ and $g(x)=\frac{x}{x-1}$. [5 marks]

(a) Find the rule and domain of $f\circ g$. [3 marks]

(b) Find the range and explain whether the composition is one-to-one. [2 marks]

2. Let $f(x)=\frac{x^2}{x^2+1}$ for $x\geq 0$. [5 marks]

(a) Find its range and prove that it is one-to-one on this domain. [2 marks]

(b) Find f^{-1}, with its domain and range. [3 marks]

3. Let $f(x) = (x-2)^2 + 1$ restricted to $x \leq 2$. [7 marks]

(a) Find its inverse, stating all necessary restrictions. [3 marks]

(b) Find every point common to the graph and its inverse. [4 marks]

4. Consider $h(x) = |x^2 - 4|$. [5 marks]

(a) State the number of distinct solutions of $h(x) = c$ for $c < 0$, $c = 0$, $0 < c < 4$, $c = 4$ and $c > 4$. [3 marks]

(b) Find the three intersections when $c = 4$ and identify the tangent intersection. [2 marks]

5. A nonzero ordinate y is transformed by $T(y)=\frac{1}{y}$. [4 marks]

(a) Find its fixed values and show that applying T twice restores every allowed ordinate. [2 marks]

(b) For $f(x)=x^2-1$, determine where $f(x)$ and $\frac{1}{f(x)}$ are both positive and where they are both negative. [2 marks]

6. Let $F(x)=\frac{x^2-2x+1}{x^2-1}$. [5 marks]

(a) Simplify and identify all holes, poles and intercepts. [3 marks]

(b) Find the range and explain why two values are missing. [2 marks]

7. Let $f(x) = \frac{x^2}{x-1}$ and consider the intersections with $y = c$. [5 marks]

(a) Derive the discriminant condition for a real intersection. [3 marks]

(b) State how many distinct intersections occur in each permitted range, including the boundary cases. [2 marks]

8. For $a > 0$, define $f_a(x) = \frac{x^2 - a^2}{x^2 + a^2}$. [5 marks]

(a) Find the zeros, stationary point, asymptote and range in terms of a. [3 marks]

(b) Describe precisely how its graph is obtained from the case $a = 1$. [2 marks]

9. A cost per unit model is $C(x) = 2 + \frac{18}{x-1}$ for $x > 1$. [5 marks]

(a) Find the batch size for cost 5 dollars. [3 marks]

(b) Explain the horizontal asymptote in context. [2 marks]

10. A student labels $(0, 0)$ as a maximum on the curve $y = \frac{x^3}{x^2-1}$. [4 marks]

(a) Use the derivative to correct the label. [3 marks]

(b) Identify two excluded inputs the sketch must respect. [1 mark]