Functions and graph sketching - Practice test 5

Name: _____ Date: _____

10 questions | 51 marks | 70 minutes

Ten multipart questions combining exact calculations, domains, graph evidence, interpretation and reasoning. Show working and label key graph features.

A scientific calculator may be used.

1. Let $f(x) = \ln x$ and $g(x) = 1 - x^2$. [5 marks]

(a) Find the domains of both compositions. [2 marks]

(b) Find and compare their ranges. [3 marks]

2. Let $f(x) = x^3 - 3x$ restricted to $[-1, 1]$. [5 marks]

(a) Justify the existence of an inverse and state its domain and range, without finding an explicit rule. [3 marks]

(b) Find the inverse images of $-2, 0, 2$. [2 marks]

3. Let $f(x) = 4 - \sqrt{2x+1}$ with its largest real domain. [5 marks]

(a) Find the inverse and its domain and range. [3 marks]

(b) Explain why the inverse formula must not be used at input 5. [2 marks]

4. Let $f(x) = x^2 - 2$. Compare $A(x) = |f(x)|$ and $B(x) = f(|x|)$. [4 marks]

(a) Find the ranges and vertical intercepts of A and B. [2 marks]

(b) Find exactly where the two graphs agree. [2 marks]

5. Let $f(x) = \frac{x}{x^2+1}$ and $r(x) = \frac{1}{f(x)}$. [5 marks]

(a) Find r, its original domain and its range. [3 marks]

(b) Explain why no real point of f is fixed by ordinate reciprocation. [2 marks]

6. Let $f(x) = \frac{x^2+ax+b}{x-1}$, where a, b are real. [5 marks]

(a) Find the condition on a, b for the graph to have a hole rather than a vertical asymptote at $x = 1$. [2 marks]

(b) If the hole is $(1, 4)$, determine a, b and the simplified rule. [3 marks]

7. Let $f(x) = \frac{x^3}{x^2 - 1}$. [6 marks]

(a) Find its asymptotes and all stationary inputs. [3 marks]

(b) Classify the stationary points and give their coordinates. [3 marks]

8. Consider the family $f_a(x) = x + \frac{a}{x}$ on $x \neq 0$. [6 marks]

(a) Find the stationary points when $a > 0$, and show that none exist when $a < 0$. [3 marks]

(b) Compare the ranges for $a = 0$ and $a < 0$. [3 marks]

9. A concentration model is $c(t)=\frac{t}{t^2+4}$ for $t\geq 0$. [6 marks]

(a) Find its maximum. [4 marks]

(b) State the long-term prediction and a limitation. [2 marks]

10. A student chooses both branches $3\pm\sqrt{x-2}$ as the inverse of $(x-3)^2+2$ restricted to $x\geq 3$. [4 marks]

(a) Correct the inverse. [2 marks]

(b) Explain why both branches cannot be used. [2 marks]