Functions and graph sketching - Practice test 4

Name: _____ Date: _____

10 questions | 48 marks | 70 minutes

Ten multipart questions combining exact calculations, domains, graph evidence, interpretation and reasoning. Show working and label key graph features.

A scientific calculator may be used.

1. Let $f(x) = \sqrt{4 - x}$ and $g(x) = \frac{1}{x}$. [5 marks]

(a) Find the domain of $f \circ g$. [3 marks]

(b) Find the domain of $g \circ f$. Why does an endpoint disappear? [2 marks]

2. Consider the family $f_a(x) = ax^2 + 2x$, with real a. [5 marks]

(a) For which a is f_a one-to-one on the entire real line? [2 marks]

(b) For $a = 1$, restrict to the increasing branch and give its inverse. [3 marks]

3. Let $f(x) = \frac{x}{x-1}$ on $\mathbb{R} \setminus \{1\}$. [4 marks]

(a) Show that f is its own inverse. [2 marks]

(b) Find its real fixed points and explain their relation to $y = x$. [2 marks]

4. Let $f(x) = |x^2 - 2x|$. [5 marks]

(a) Give its piecewise rule and all local extrema. [3 marks]

(b) For which real c does $f(x) = c$ have four distinct solutions? [2 marks]

5. Let $r(x) = \frac{1}{x^2+2x+2}$. [5 marks]

(a) Find its domain, symmetry, maximum and range. [3 marks]

(b) Find all intersections of the graph with $y = \frac{1}{2}$. [2 marks]

6. For $f(x) = \frac{x^3-x}{x^2-1}$: [4 marks]

(a) Simplify, state the domain, and identify every hole. [2 marks]

(b) Find the range and decide whether the function is one-to-one. [2 marks]

7. Let $f(x) = \frac{x^2-1}{x^2+1}$. [5 marks]

(a) Find its intercepts, stationary point and range. [3 marks]

(b) How many intersections with a horizontal line $y = c$ occur for $-1 < c < 1$? Justify. [2 marks]

8. A rational graph has rule $y = x + \frac{4}{x-1}$. [6 marks]

(a) Find its asymptotes and stationary points. [3 marks]

(b) Justify its range using a quadratic discriminant. [3 marks]

9. The height model $h(x) = |x - 4| + 2$ is used on $0 \le x \le 8$. [5 marks]

(a) Find the minimum and its location. [2 marks]

(b) For which inputs is height 5? [3 marks]

10. A graph of $\frac{x}{x^2+1}$ is said never to cross $y = 0$ because $y = 0$ is its asymptote. [4 marks]

(a) Refute the claim. [2 marks]

(b) State what a horizontal asymptote actually describes. [2 marks]