Functions and graph sketching - Practice test 2

Name: _____ Date: _____

10 questions | 48 marks | 70 minutes

Ten multipart questions combining exact calculations, domains, graph evidence, interpretation and reasoning. Show working and label key graph features.

A scientific calculator may be used.

1. Let $f(x) = \frac{1}{x-1}$ and $g(x) = \sqrt{x}$. [4 marks]

(a) Find $(f \circ g)(x)$ and its domain. [2 marks]

(b) Find $(g \circ f)(x)$ and its domain. [2 marks]

2. Consider $f(x) = (x-1)^2$ on the restricted domain $[-1, 1]$. [5 marks]

(a) Prove that this restriction is one-to-one. [2 marks]

(b) Find the inverse, including its domain and range. [3 marks]

3. Let $f(x) = \frac{2x+3}{x-1}$, $x \neq 1$. [4 marks]

(a) Find its inverse and excluded inverse input. [2 marks]

(b) Find $f^{-1}(-3)$ and verify the result in the original rule. [2 marks]

4. Let $v(x) = |x^2 - 9|$. [5 marks]

(a) Find the zeros, local extrema and range. [3 marks]

(b) Find all intersections with $y = 5$. [2 marks]

5. Let $r(x) = -\frac{3}{x+2} + 1$. [5 marks]

(a) Find its asymptotes, centre of symmetry and intercepts. [3 marks]

(b) Determine where the graph is increasing and explain its range. [2 marks]

6. Let $f(x) = \frac{x+2}{x^2+x-2}$. [5 marks]

(a) Factor, simplify and locate every discontinuity. [3 marks]

(b) Find the range, taking account of the hole. [2 marks]

7. Let $f(x) = \frac{x^2}{x+2}$. [5 marks]

(a) Use division to find its asymptotes. [2 marks]

(b) Find and classify its stationary points. [3 marks]

8. Let $f(x) = \frac{x^2+1}{x^2+4}$. [5 marks]

(a) Find its symmetry, stationary point and range. [3 marks]

(b) Give all intercepts and asymptotes needed for a sketch. [2 marks]

9. A machine is calibrated by $y = (x-4)^2 + 1$ on $x \geq 4$. [5 marks]

(a) Find the inverse calibration and its domain. [3 marks]

(b) Find the input producing output 10. [2 marks]

10. A sketch of $\frac{1}{x^2+1}$ shows two vertical asymptotes. [5 marks]

(a) Explain why that cannot be correct. [2 marks]

(b) State its actual domain, range and end behaviour. [3 marks]