Functions and graph sketching - Practice test 1

10 questions | 46 marks | 70 minutes

Ten multipart questions combining exact calculations, domains, graph evidence, interpretation and reasoning. Show working and label key graph features.

Equivalent correct methods earn the corresponding marks.

1. Let $f(x)=\sqrt{x+2}$ and $g(x)=3x-1$. [4 marks]

(a) Find $(f\circ g)(x)$ and its domain. [2 marks]

The inner line is defined for all real inputs. The outer root requires $3x-1+2\geq 0$.

Answer: $\sqrt{3x+1}$, $x\geq -\frac{1}{3}$.

(b) Find $(g\circ f)(x)$ and its domain. Are the two functions equal? [2 marks]

The root now acts first and requires $x\geq -2$. The rules and domains differ.

Answer: $3\sqrt{x+2}-1$, $x\geq -2$; they are not equal.

2. Consider $f(x)=x^2-4x+1$ on the real numbers. [4 marks]

(a) Use two distinct inputs to show that f is not one-to-one. [2 marks]

The inputs 1 and 3 are both allowed and unequal.

Answer: For example, $f(1)=f(3)=-2$.

(b) Give the largest interval starting at the vertex on which f is increasing, and state its range. [2 marks]

Complete the square: $f(x)=(x-2)^2-3$. The right branch increases from its minimum.

Answer: Domain $[2,\infty)$; range $[-3,\infty)$.

3. Let $f(x) = 5 - 2x$ on the real numbers. [4 marks]

(a) Find f^{-1} and verify one of the inverse compositions. [2 marks]

From $y = 5 - 2x$, solve $x = \frac{5-y}{2}$. Substitute: $5 - 2(\frac{5-x}{2}) = x$.

Answer: $f^{-1}(x) = \frac{5-x}{2}$; $f(f^{-1}(x)) = x$.

(b) Find the point where the graph of f meets the graph of its inverse. [2 marks]

Solve the two line equations together: $5 - 2x = \frac{5-x}{2}$ gives $x = \frac{5}{3}$, and substitution gives $y = \frac{5}{3}$.

Answer: $(\frac{5}{3}, \frac{5}{3})$.

4. Let $h(x) = |2x + 2| - 4$. [5 marks]

(a) Write h piecewise and find its vertex. [2 marks]

The expression inside the absolute value changes sign at $x = -1$.

Answer: $h(x) = -2x - 6$ for $x < -1$, and $h(x) = 2x - 2$ for $x \geq -1$; vertex $(-1, -4)$.

(b) Find the intercepts and solve $h(x) < 0$. [3 marks]

$|2x + 2| = 4$ gives $x = -3, 1$; substitute zero for the vertical intercept. The graph is below zero between its roots.

Answer: Intercepts $(-3, 0), (1, 0), (0, -2)$; $-3 < x < 1$.

5. Let $r(x) = \frac{1}{(x-2)^2}$. [5 marks]

(a) State its domain, asymptotes and range. [3 marks]

The squared denominator is positive away from 2, can be arbitrarily small, and grows without bound.

Answer: Domain $x \neq 2$; asymptotes $x = 2$, $y = 0$; range $(0, \infty)$.

(b) State its symmetry and solve $r(x) = 1$. [2 marks]

Inputs equally far from 2 have equal squares. Solve $(x-2)^2 = 1$.

Answer: Symmetric about $x = 2$; $x = 1$ or $x = 3$.

6. Consider $q(x) = \frac{x^2 - 9}{x - 3}$. [4 marks]

(a) Simplify the rule while retaining the original domain. [2 marks]

Factor the numerator as $(x - 3)(x + 3)$; cancellation is valid only when $x - 3 \neq 0$.

Answer: $q(x) = x + 3$ for $x \neq 3$.

(b) Give the hole and both intercepts. [2 marks]

The missing input is 3, whose line value would be 6. The root -3 and input 0 are both permitted.

Answer: Hole $(3, 6)$; intercepts $(-3, 0)$ and $(0, 3)$.

7. For $f(x) = x + \frac{4}{x}$, with $x \neq 0$: [6 marks]

(a) Find and classify the stationary points. [3 marks]

$f'(x) = 1 - \frac{4}{x^2}$. The derivative is positive for $|x| > 2$ and negative for $0 < |x| < 2$.

Answer: Local maximum $(-2, -4)$; local minimum $(2, 4)$.

(b) Give the asymptotes and justify the complete range. [3 marks]

For $x > 0$, $x + \frac{4}{x} \geq 4$, and the positive branch attains every value from 4 upward. Odd symmetry gives the negative branch.

Answer: Asymptotes $x = 0$, $y = x$; range $(-\infty, -4] \cup [4, \infty)$.

8. Let $u(x)=\frac{1}{x^2+4}$. [5 marks]

(a) Use differentiation and symmetry to locate its maximum. [2 marks]

$u'(x)=-\frac{2x}{(x^2+4)^2}$, positive for $x<0$ and negative for $x>0$.

Answer: Global maximum $(0,\frac{1}{4})$; the graph is even.

(b) Describe a complete sketch, including intercepts and end behaviour. [3 marks]

The denominator is never zero, so there is no vertical asymptote. Label the maximum and the horizontal asymptote.

Answer: No horizontal-axis intercept; vertical intercept $\frac{1}{4}$; all real inputs; positive branches approach $y=0$ at both ends.

9. A sensor uses output $f(t)=\sqrt{t-2}$ for time $t\geq 2$. A clock display reports $t=3s+2$. [5 marks]

(a) Express the output in s and give its domain. [3 marks]

Require $3s+2\geq 2$.

Answer: $\sqrt{3s}$, $s\geq 0$.

(b) Find the displayed s when output is 6. [2 marks]

$\sqrt{3s}=6$ gives $3s=36$.

Answer: $s=12$.

10. A student states that $\frac{x^2-1}{x-1}$ and $x+1$ are the same function on all real inputs. [4 marks]

(a) Identify the false part of the claim. [2 marks]

The original denominator is zero there.

Answer: The quotient excludes $x=1$.

(b) Describe the graphical difference. [2 marks]

The line includes this point.

Answer: The quotient has a hole at $(1,2)$.