Combining graphical/algebraic/calculus information in sketches

6 questions | 23 marks

Equivalent correct methods earn the corresponding marks.

1. Sketch $y = x + \frac{1}{x}$ using its domain, extrema and asymptotes. [5 marks]

The equation is undefined at 0.

The difference from $y = x$ is $\frac{1}{x}$, tending to zero at either end.

Derivative signs determine each branch shape.

Answer: Two odd-symmetric branches; $x = 0$ is vertical, $y = x$ is oblique; extrema $(-1, -2)$ and $(1, 2)$; no intercepts.

2. Give a complete sketch description of $y = |x - 2| - 3$. [4 marks]

Use the two linear branches and place the exact intercepts.

The function is continuous but not differentiable at $x = 2$.

Answer: V shape with vertex $(2, -3)$, roots -1 and 5, y-intercept -1; slopes -1 then 1; range $[-3, \infty)$.

3. A student draws one continuous curve through $x = 1$ for $y = \frac{x^2}{x - 1}$. Explain and correct the sketch. [3 marks]

The original denominator is zero at 1.

The numerator is nonzero there, giving a pole rather than a hole.

Answer: The graph must be broken into branches separated by the vertical asymptote $x = 1$.

4. For $y = \frac{1}{x^2+1}$, justify the complete graph from symmetry, derivative and limits. [4 marks]

$y' = -\frac{2x}{(x^2+1)^2}$.

The derivative is positive for negative x and negative for positive x.

The denominator never vanishes.

Answer: An even positive curve, maximum $(0, 1)$, approaching $y = 0$ at both ends with no vertical asymptote.

5. Two formulas simplify to the same expression. Explain why their graphs might still differ. [3 marks]

Compare $x + 1$ with $\frac{x^2-1}{x-1}$.

Only the second excludes $x = 1$.

Answer: Their original domains may differ, producing a hole in one graph.

6. Find the tangent to $f(x) = \frac{x}{x^2+1}$ at $x = 0$ and explain why it does not determine the whole graph. [4 marks]

$f(0) = 0$ and $f'(0) = 1$.

A tangent is local information; extrema and end behaviour need separate analysis.

Answer: $y = x$.